H.P. Paar **PHYS 4C: Electrmagnetism** Fall 2015

Quiz # 4 solutions
Solutions by Yury Kiselev

1. (10 points)
   The dielectric slab is neutral and the electric field is uniform, so total force $\vec{F} = \int \vec{E}\ \mathrm{d}q = \vec{E} Q_{tot} = 0$.

2. (30 points)

   (a) (6 points) Voltage across the capacitor is V, so $q = CV = \frac{\epsilon_0 A}{d} V$. The energy stored in the capacitor is $E = \frac{1}{2} CV^2 = \frac{\epsilon_0 A}{2d} V^2$.

   (b) (6 points) Voltage across the capacitor is still V, so $q_k = C_k V = \frac{\epsilon_0 k A}{d} V$ and the stored energy is $E_k = \frac{1}{2} C_k V^2 = \frac{\epsilon_0 k A}{2d} V^2$.

   (c) (6 points) Both battery and external force inserting the dielectric are doing work to change the energy stored in the capacitor. The charge changes because the capacitor is connected to the battery while his capacitance changes.

   (d) (6 points) Battery delivers energy $W_b = V \Delta q = \frac{\epsilon_0 (k-1) A}{d} V^2$. Usually $k > 1$ and the charge on the plate of capacitor increases, so the battery delivers positive amount of energy.

   (e) (6 points) Person delivers the rest of the energy difference between the initial and final amounts of stored energy in the capacitor. $W_p = E_k - E - W_b = \frac{\epsilon_0 (k-1) A}{2d} V^2 - \frac{\epsilon_0 (k-1) A}{d} V^2 = -\frac{\epsilon_0 (k-1) A}{2d} V^2$. So the person is doing negative work.

3. (20 points) Due to the symmetry and positively charged plane, the electric field will be directed along z axis. It will be pointing up for $z > 0$, and down for $z < 0$. Also, it should be symmetric relative to $z = 0$ plane: $\vec{E}(x, y, z) = -\vec{E}(x, y, -z)$. Now let's choose Gauss' surface: it will be a cylinder oriented along z axis, with symmetrical (relative to $z = 0$) positions of up and down bases: they will lie in z and $-z$ planes. There are two cases: cylinder height is bigger than d ($z > d/2$) and less than d. Gauss' law says:

   $$\Phi = \frac{Q_{enc}}{\epsilon_0}.$$

   Here only electric field through the bases will contribute to the flux, $\Phi = 2EA$, where A – is the area of a base of the cylinder. Enclosed charge depends on z. If $z < d/2$:

   $$Q_{enc}(z < d/2) = A \int_{-z}^{z} \rho(z)\ \mathrm{d}z = 2A \int_{0}^{z} \alpha z\ \mathrm{d}z = A \alpha z^2.$$

   If $z \geq d/2$: $Q_{enc}(z \geq d/2) = A\alpha (d/2)^2$.

So, the electric field inside the slab is $E(|z| < d/2) = \frac{Q_{enc}}{2A\epsilon_0} = \frac{\alpha z^2}{2\epsilon_0}$ and magnitude of the electric field outside is $E(|z| \geq d/2) = \frac{\alpha d^2}{8\epsilon_0}$. The direction of the electric field is up along z axis for $z > 0$ and in the negative direction of the z axis for $z < 0$.

4. (20 points)

   (a) (5 points) The electric field is radial, so the potential difference between points A and B does not depend on their angular positions, but only on the distances from the origin: r_1 and r_2. Let's assume, then, that they are on the same radius vector.
   $$\Delta V = -\int_{r_1}^{r_2} \vec{E}\ \mathrm{d}\vec{r} = -\int_{r_1}^{r_2} \beta r\ \mathrm{d}r = -\beta(r_2^2 - r_1^2)/2.$$

   (b) (5 points) If points A and B are not at the same radius vector, the result is the same. We can prove that by moving from A to the point on the spherical shell r_2, where point B resides, going along the radius vector, defined by point A. Later we will move from that point on shell r_2 to point B. Along this path $\mathrm{d}\vec{r} \perp \vec{E}$, so the second potential difference contribution will be zero and the answer does not depend on the angular position of points A and B.

   (c) (5 points) Electric field is pointing radially out, so the electron will move radially in, because its charge is negative.

   (d) (5 points) Total energy is conserved, so the kinetic energy at r_1 will be $K = |e|\beta(r_2^2 - r_1^2)/2$.

5. (20 points)

   (a) (15 points) Those currents produce magnetic fields going around them (according to the right-had rule). In the $z = 0$ plane both magnetic fields have only z components. The result is a superposition of B_1^z and B_2^z, where $B_1^z = \pm\frac{\mu_0}{2\pi r_1} i_1$ and $B_2^z = \pm\frac{\mu_0}{2\pi r_2} i_2$, where r_1 and r_2 are distances from the x and y axes; plus and minus signs correspond to out of and into the plane directions. Let's take a look at some point (x, y) in $z = 0$ plane. The component B_1^z is positive if $y > 0$ and negative if $y < 0$, so we can write down its projection to z axis as $B_1^z(x, y) = \frac{\mu_0}{2\pi y} i_1$, because $r_1 = |y|$.
   By the same logic, $B_2^z(x, y) = \frac{\mu_0}{2\pi(-x)} i_2$, because $r_2 = |x|$ and projection to z axis is positive for $x < 0$ and negative for $x > 0$. The sum is
   $$B_z(x, y) = \frac{\mu_0}{2\pi y} i_1 - \frac{\mu_0}{2\pi x} i_2.$$

   (b) (5 points) Setting $B_z = 0$ gives us $i_1/y = i_2/x$, so $y = (i_1/i_2)x$ – this is an equation of a line along which the total magnetic field is zero.