

Quiz # 2 solutions
Solutions by Yury Kiselev

1. (15 points)

(a) (8 points) Electric field components are

$$\begin{aligned} E_x &= -\frac{\partial V}{\partial x} = -\alpha 2x \ln(\lambda z) \\ E_y &= -\frac{\partial V}{\partial y} = -\alpha 2y \ln(\lambda z) \\ E_z &= -\frac{\partial V}{\partial z} = -\alpha \frac{(x^2+y^2)}{z} > 0, \end{aligned}$$

where $\alpha < 0$, so $-\alpha = |\alpha|$.

So, E_z depends on three variables, but we can plot it versus $r_{xy} = \sqrt{x^2 + y^2}$ and z due to the rotational symmetry around z axis. We will not include the actual plot, as it's pretty obvious.

(b) (7 points)

Let's assume the electron is placed at point (x, y, z) , and its initial speed is zero. The acceleration will be in the opposite direction of electric field, because charge of the electron is negative.

$$\begin{aligned} a_x &= \beta 2x \ln(\lambda z) \\ a_y &= \beta 2y \ln(\lambda z) \\ a_z &= \beta \frac{(x^2+y^2)}{z}, \end{aligned}$$

where constant $\beta = -|\alpha| \cdot |e|/m_e$. At first moment the direction of motion coincides with the direction of the acceleration, but later the direction of motion will deviate from the acceleration, because of the nonzero velocity electron will acquire.

The electron will end up in the minimum of potential energy position, which is $U = |e||\alpha|(x^2 + y^2) \ln(\lambda z)$. The potential energy minimum is at $z = 0$ and $r_{xy} = \sqrt{x^2 + y^2} = \infty$, so the electron will end up in that region.

2. (15 points)

(a) (5 points) Charge on the right plates of C_1 is opposite to the charge on the left plate of the capacitor C_2 due to the fact that the portion of the wire between the plates is not connected to rest of the system, so total charge remains zero inside. So, $q_1 = q_2 = q$ and the total voltage is $V = q_1/C_1 + q_2/C_2 = q(1/C_1 + 1/C_2)$, so the charge on the capacitors is

$$q = \frac{VC_1C_2}{C_1+C_2}.$$

The terminal B is positive, so the positive current flows from B to A , which means that the right plate of C_2 has charge q , the left plate of C_2 then has charge $-q$, so the right plate of C_1 has charge q and the left plate of C_1 has charge $-q$.

(b) (5 points)

$$V_1 = q/C_1 = \frac{VC_2}{C_1+C_2},$$

$$V_2 = q/C_2 = \frac{VC_1}{C_1+C_2}.$$

(c) (5 points)

$$U_1 = \frac{1}{2} C_1 V_1^2 = \frac{1}{2} \frac{V^2 C_1 C_2^2}{(C_1+C_2)^2},$$

$$U_2 = \frac{1}{2} C_2 V_2^2 = \frac{1}{2} \frac{V^2 C_1^2 C_2}{(C_1+C_2)^2},$$

so the total potential energy is

$$U = U_1 + U_2 = \frac{1}{2} \frac{V^2 C_1 C_2}{C_1+C_2}.$$

3. (35 points) To calculate the effective field in the presence of dielectric, we need to find the field without dielectric and divide by the dielectric constant κ .

(a) (10 points) Electric field contributions from the outer shell are cancelled in the region $r < R_2$ (can prove it by Gauss' theorem), so the electric field without the dielectric is the same as from a point charge Q located at the center: $E_w = \frac{kQ}{r^2}$ and with the dielectric it will be $E = \frac{kQ}{\kappa r^2}$.

(b) (5 points) To find the electric field outside we will use Gauss' law: the total charge inside $Q - q$, because total charge of the dielectric is zero. The electric field is then $E = \frac{k(Q-q)}{r^2}$.

(c) (10 points) The potential relative to infinity $V = - \int_{\infty}^{R_2} E \, dr = \int_{R_2}^{\infty} E \, dr =$
 $k(Q - q) \int_{R_2}^{\infty} \frac{1}{r^2} \, dr = k(Q - q) \left(\frac{-1}{\infty} - \frac{-1}{R_2} \right) = \frac{k(Q-q)}{R_2}.$

(d) (10 points) The potential of the inner shell relative to infinity is $V = - \int_{\infty}^{R_1} E \, dr =$
 $\int_{R_1}^{\infty} E \, dr = \int_{R_1}^{R_2} E \, dr + \int_{R_2}^{\infty} E \, dr.$ The second integral we calculated in part (c).
 The first one is: $I_1 = \int_{R_1}^{R_2} E \, dr = \int_{R_1}^{R_2} \frac{kQ}{\kappa r^2} \, dr = \frac{kQ}{\kappa} \left(\frac{-1}{R_2} - \frac{-1}{R_1} \right).$ So the potential
 of the inner shell is $V_1 = \frac{k(Q-q)}{R_2} + \frac{kQ}{\kappa} \left(\frac{1}{R_1} - \frac{1}{R_2} \right).$

4. (35 points)

- (a) (15 points) The electric field around the wire without the dielectric may be found via Gauss' law: $Q_{in} = \lambda L$, where L is a length of a cylinder surface around the wire. $\Phi = E_w \cdot L2\pi r$, so $E_w = \frac{\lambda}{2\pi\epsilon_0 r}$. The field with the dielectric is $E = E_w/\kappa = \frac{\lambda}{2\pi\epsilon_0 \kappa r}$. The field inside conducting wire is, obviously, zero.
- (b) (10 points) No dielectric, so the bare formula works here: $E = E_w = \frac{\lambda}{2\pi\epsilon_0 r}$.
- (c) (10 points) Potential relative to infinity will be infinity, as integral of $1/r$ is diverging.