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Fall 2015

 $\frac{\text{Quizz } \# 1 \text{ solutions}}{\text{Solutions by Yury Kiselev}}$

1. (20 points) We will assume that the first charge q_1 is positive. The negative q_1 would result in the opposite sign for charge q_2 .

The electric field at point P is

$$E_x = k \frac{q_1}{a^2} - k \frac{q_2}{b^2} = 0,$$

so $q_2 = q_1 \cdot (b/a)^2$. It has the same sign as q_1 . We assumed it to be positive.

- 2. (20 points) In my coordinate system we will have electron moving in positive direction of X axis and electric field will be pointing in the positive direction of Y axis.
 - (a) (7 points) Using second Newton's law,

$$m_e a_y = eE < 0,$$

because charge of the electron is negative. Other components of acceleration are zero. So, the direction of acceleration is in the negative direction of the Y axis. The acceleration is $a_y = eE/m_e < 0$; $a_x = a_z = 0$.

- (b) (7 points) This motion is a motion in the constant field with constant acceleration, pointing down. We know this type of motion very well it's the same as free fall motion in gravitational field of the Earth. We can say that this is ballistic motion, or parabolic motion, as the trajectory of the electron will be a parabola.
- (c) (6 points) Initial velocity of the electron is $\boldsymbol{v}_0 = (v_0, 0)$. The final velocity after time t will be $\boldsymbol{v}_f = (v_0, teE/m_e)$. The y-component is negative because e < 0. The change in the kinetic energy is then $\Delta(m_e v^2/2) = (teE)^2/(2m_e)$.
- 3. (20 points)
 - (a) (7 points) The observer prevents dipole from accelerating (meaning angular acceleration of rotation), while the potential energy of the dipole decreases: $U_i = -pE = 0$ and $U_f = -pE = -pE$. The change of kinetic energy of dipole is a sum of a work of all forces: $\Delta K = \Sigma W_f = W_E + W_{obs}$, which is a work of the electric field plus a work of the observer. $\Delta K = 0$, so $W_{obs} = -W_E = -(-\Delta U) = \Delta U = -pE$. Work of the observer is negative.
 - (b) (7 points) As we already found, $W_E = -\Delta U = pE$.
 - (c) (6 points) Potential energy depends only on the orientation of the dipole, so the work to translate the dipole is zero. It is related to the fact that the total charge of a dipole is zero.

- 4. (20 points) In each part of the problem we will draw an imaginary sphere with radius r and use the Gauss law. Our problem is spherically symmetric, so the electric field may be pointing only perpendicular to the drawn sphere and should be constant in magnitude on the points of the drawn sphere. The total flux then can be expressed as $\Phi = EA = E4\pi r^2 = Q_{inside}/\epsilon_0$.
 - (a) (4 points) We will draw a sphere with $r < R_1$. The total charge inside the drawn sphere is Q, so the flux should will be $\Phi = Q/\epsilon_0 = EA$ and $E = Q/(4\pi\epsilon_0 r^2)$.
 - (b) (4 points) When $R_1 < r < R_2$, a part of the charged insulating material will be inside the drawn sphere. Let's calculate the volume of the insulating material inside: $V = \frac{4}{3}\pi r^3 - \frac{4}{3}\pi R_1^3$, so the charge inside is $Q_{in} = Q + \rho V = Q + \rho (\frac{4}{3}\pi r^3 - \frac{4}{3}\pi R_1^3)$. The electric field is $E = Q_{in}/\epsilon_0/A = [Q + \rho(\frac{4}{3}\pi r^3 - \frac{4}{3}\pi R_1^3)]/(\epsilon_0 4\pi r^2) = [3Q/4\pi + \rho(r^3 - R_1^3)]/(3\epsilon_0 r^2)$. The direction of the electric field is radially outward from the center of the construction, assuming the charge density and charge Q are both positive. In the case they are both negative, the electric field will be radially inward.
 - (c) (4 points) For $r > R_2$, $Q_{in} = Q + \rho(\frac{4}{3}\pi R_2^3 \frac{4}{3}\pi R_1^3)$, so the electric field is $E = Q_{in}/\epsilon_0/A = [Q + \rho(\frac{4}{3}\pi R_2^3 \frac{4}{3}\pi R_1^3)]/(\epsilon_0 4\pi r^2) = [3Q/4\pi + \rho(R_2^3 R_1^3)]/(3\epsilon_0 r^2)$. The direction is the same as in part (b).
 - (d) (4 points) Yes, it is. At the boundary between I and II the electric field at left side is $Q/(4\pi\epsilon_0R_1^2)$, and the right it's $E_{R1+} = [3Q/4\pi + \rho(r^3 R_1^3)]/(3\epsilon_0r^2) = Q/(4\pi\epsilon_0R_1^2)$, as $r = R_1$. At the next boundary: $E_{R2-} = [3Q/4\pi + \rho(R_2^3 - R_1^3)]/(3\epsilon_0R_2^2) = E_{R2+}$, as $r = R_2$,

At the next boundary: $E_{R2-} = [3Q/4\pi + \rho(R_2^3 - R_1^3)]/(3\epsilon_0R_2^2) = E_{R2+}$, as $r = R_2$, so the electric field is continuous.

- (e) (4 points) Fig. 1 (page 3) is the plot for $R_1 = 1, R_2 = 2, \epsilon_0 = \rho = 1, Q = 4\pi/3$.
- 5. (20 points) The total flux equals to the flux through four sides plus flux through two small distant sides with area d^2 . The flux through the latter two can be neglected, because they are further away and their area is d^2 , which is much less than $L \cdot d$ area of the other sides. By symmetry, the other other four sides yield same contribution to the total flux, so the electric flux through one of them is $\Phi_1 = \Phi_t/4 = Q/(4\epsilon_0)$.
- 6. (30 points) The force on charge Q will be $F_x = k \frac{Qq_1}{(a+\Delta x)^2} k \frac{Qq_2}{(b-\Delta x)^2} = k \frac{Qq_1}{(a+\Delta x)^2} k \frac{b^2}{a^2} \frac{Qq_1}{(b-\Delta x)^2}$.

To simplify this expression we will use that Δx is small, so

 $\begin{array}{l} 1/(c+d\Delta x)^2 &= (1/c^2)(1/(1+d\Delta x/c))^2 \approx (1/c^2)(1-2d\Delta x/c), \mbox{ so } 1/(a+\Delta x)^2 \approx (1/a^2)(1-2\Delta x/a) \mbox{ and } 1/(b-\Delta x)^2 \approx (1/b^2)(1+2\Delta x/b). \mbox{ Using these expressions,} \\ F_x &= k \frac{Qq_1}{a^2}(1-2\Delta x/a) - k \frac{b^2}{a^2} \frac{Qq_1}{b^2}(1+2\Delta x/b) = k Qq_1 \frac{1}{a^2}(1-2\Delta x/a-1-2\Delta x/b) = -k Qq_1 \frac{2\Delta x}{a^2}(1/a+1/b). \mbox{ Note that this expression equals to zero when } \Delta x = 0. \mbox{ Also,} \\ \mbox{it's negative if } \Delta x > 0 \mbox{ and vice versa, so this looks like a returning force of a harmonic oscillator.} \end{array}$

