December 9, 2015 Hans P. Paar

4C FINAL

This Final is closed book.

Make sure to always indicate positive directions and make large and neat figures. Specify the units of numerical answers.

The electron's mass $m = 9.11 \times 10^{-31}$ Kg and its charge $|e| = 1.6 \times 10^{-19}$ C, $1/(4\pi\epsilon_0) = 9 \times 10^9$ C/(Nm²), $\mu_0 = 4\pi \times 10^{-7}$ Tm/A, the magnetic moment of a loop of current is given by $\mu = \mu_0 iA$ with the direction of μ given by the RH rule applied to the current in the loop.

PROBLEM 1 (20 points)

Write down the four equations of Maxwell in differential and integral form. Group them in pairs where each member of a pair represents the same physics in different notation.

PROBLEM 2 (30 points)

Two infinitely long one dimensional charge distributions are positioned along the x-axis and the y-axis. The charge distributions are uniform and their charge densities are specified by $\lambda < 0$. Calculate the electric field everywhere in the x, y plane except on the coordinate axes.

PROBLEM 3 (30 points)

An electric dipole is positioned in an inhomogeneous electric field $\boldsymbol{E} = \alpha x \boldsymbol{e}_x$ with $\alpha > 0$ a constant. The dipole has a length ℓ and a dipole moment \boldsymbol{p} . The vector \boldsymbol{p} makes an angle β with \boldsymbol{E} .

- a. Calculate the total force acting on the dipole.
- b. Calculate the total torque on the dipole.
- c. In view of your answers in a) and b), what is going to happen to the dipole?

PROBLEM 4 (20 points)

An electric potential V is given by $V = \alpha/r^2$ where $\mathbf{r} = (x, y, z)$ and α is a constant. Calculate the resulting electric field.

PROBLEM 5 (30 points)

An line of charges is positioned along the x-axis between x = 0 and x = L. The linear charge density $\lambda < 0$ is constant. A point P is located on the x-axis at position x > L.

- a. Calculate the electric potential at point P.
- b. Calculate the electric field at point P.
- c. Consider the limit x >> L in your answers of a) and b). Do these two limts make sense? Explain.
- d. There is a relation between electric potential and electric field at pointP. What is it and do your answers in a) and b) satisy it?

PROBLEM 6 (30 points)

A dielectric with dielectric constant κ is located between the plates of a planar capacitor. It fills the space between them. With the dielectric in place the capacitance is C. The plates of the capacitor carry charges of +Q and -Q respectively. The capacitor is not connected to anything. An observer pulls out the dielectric.

How much work does she have to do pulling it out? Carefully state whether that work is positive or negative.

PROBLEM 7 (20 points)

A beam of electrons of velocity v directed along the +x-axis enters a region with a uniform magnetic field in the +z-direction. This causes the electron's trajectory to deviate from the direction of the x-axis.

a. What kind of trajectory do the electrons follow when they are in the region of the magnetic field? Specify fully their trajectory, velocity, direction, etc.

b. An observer decides to correct the deviation of the trajectory by applying an electric field in addition to the magnetic field. In which direction should the electric field be and how large should it be so that the electrons continue in a straight line when they enter the region of the magnetic field?

PROBLEM 8 (20 points)

Two parallel conductors (rails) at a distance ℓ have a conducting bar of mass m across it at 90 deg as shown in the figure. The rails are connected to a DC power supply through a switch as shown. The battery (power supply) delivers a DC current I. The battery (power supply) will maintain the current I despite the induced voltage caused by the movement of the bar. There is a magnetic field \boldsymbol{B} perpendicular to the rails. When the switch is closed the power supply will provide its current instantaneously.

- a. Calculate the force on the bar after the switch is closed.
- b. What is the velocity of the bar after a time t?
- c. How much energy has the power supply delivered at time t?

PROBLEM 9 (30 points)

A magnetic field is associated with a vector poential A(x, y, z) given by $A = i xy e_z$ where x and y are spatial coordinates and i is a constant.

- a. Calculate the magnetic field.
- b. Calculate the magnitude of the magnetic field and specify its direction.
- c. Calculate $\boldsymbol{\nabla} \cdot \boldsymbol{A}$.
- d. We have defined Gauge Transformations in order to be able to make $\nabla \cdot \mathbf{A} = 0$. Why did we want that condition to be true?

PROBLEM 10 (20 points)

People who want to measure a magnetic field sometimes use the flip-coil method. They use a coil consisting of N turns, each of area A. Initially the plane of the turns is perpendicular the the unknown magnetic field. They then rotate the coil over 180 deg with angular veocity ω and measure the induced voltage in the coil.

- a. What is the voltage as function of time?
- b. Calculate the magnetic field from the measured voltage.

PROBLEM 11 (20 points)

Two coils with self inductances L_1 and L_2 respectively are connected in parallel as shown in the figure.

Calculate the equivalent self inductance of the single coil that replaces the pair.

PROBLEM 12 (30 points).

An inductance, a capacitance, and a switch are connected in a circuit as shown in the Figure. The inductance is L and the capacitance is C. Initially the capacitor is charged and has charges $\pm Q_0$ on its plates. At t = 0 the switch is closed.

- a. Calculate the differential equation that governs the circuit. Let the dependent variable be the current in the circuit i(t).
- b. Solve the differential equation for i(t).
- c. Calculate Q(t) from your answer in b).
- d. Plot the current in the circuit and the charge on the capacitor as function of time.

EXTRA CREDIT (30 points)

- a. Given a current density j(r) write the expression for the resulting vector potential A(r).
- b. Calculate $\nabla \cdot A$ using A from a).