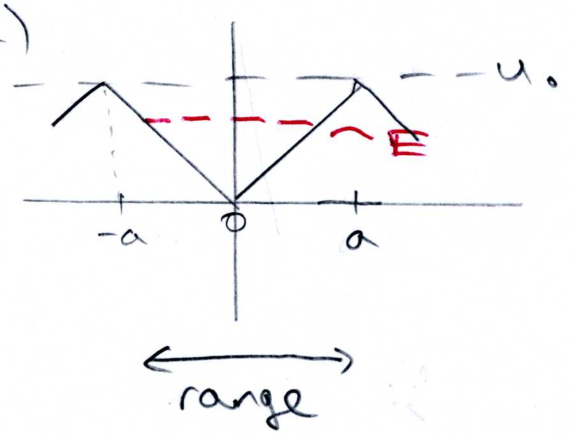
Physics 4A - Quiz #4

1 (a)



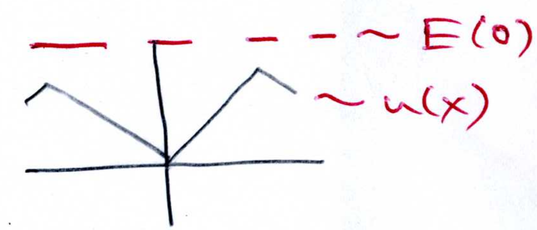
If $K < U_0$, particle is constrained to move between $x=-a$ & $x=a$.
In other words, $|x|<a$.

Remember, $E = K + U$ & we are given $E(0) = K(0) + U(0)$
$\Rightarrow E(0) = K(0)$.

(b) particle will oscillate back & forth

(c) $E(0) = K(0) + U(0)$
$\Rightarrow E(0) = K(0)$.

If $K(0) > U_0 \Rightarrow E(0) > U_0$



Since $E > U_0$, particle will move in a straight line, and it will decelerate moving "up" the peak & accelerate moving "down" the peak.
E is conserved, so accelerations are cyclic.

(d) Find velocity @ $x=a$ for $E>U_0$:

$E = K + U$

$\Rightarrow E - U = \frac{1}{2}mv^2$

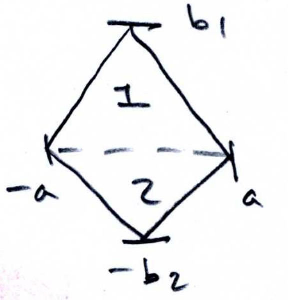
or, $v = \sqrt{\frac{2}{m}(E-U)}$

$\rightarrow v(x=a) = \sqrt{\frac{2}{m}(E-U(a))}$

$= \sqrt{\frac{2}{m}(E-U_0)}$

(e) Velocity @ all the peaks are the same. Again, this is due to the conservation of energy.

2| Let's label the disk by regions:



If we assume the plate sits @ $z=0$ & extends in the $+\hat{k}$ direction, then

$$\vec{z}_{com} = +\frac{D}{2}\hat{k}$$

If you choose to have the middle of the plate @ $z=0$, the again by symmetry we know that:

$$\vec{z}_{com} = 0\,\hat{k}$$

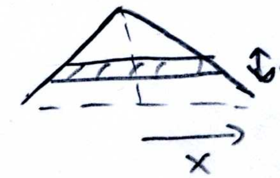
I'll choose the first of these.

To find $\vec{x}_{com}$, we can again exploit symmetry to find:

$$\vec{x}_{com} = 0\,\hat{i}$$

This just leaves $\vec{y}_{com}$...

We can also simplify the calculation of $\vec{y}_{com}$ because the plate is "homogeneous"



$$\frac{dm}{dA} = \rho \equiv \text{constant}$$

$$\rightarrow dm = 2x\rho\, dy$$

The equation for y is:

$$y = -\frac{b_1}{a}(x-a) \Rightarrow x = a\left(1-\frac{1}{b_1}y\right)$$

So, $$y^{(1)}_{com} = \frac{1}{M}\int y\, dm \equiv \text{com in (1)}$$

$$= \frac{1}{\rho A}\int_{y=0}^{y=b_1} y \cdot 2a\left(1-\frac{1}{b_1}y\right)\rho\, dy$$

$$= \frac{1}{ab_1}\cdot 2a\int_0^{b_1}\left(y-\frac{1}{b_1}y\right)dy$$

$$= \frac{1}{3}b_1$$

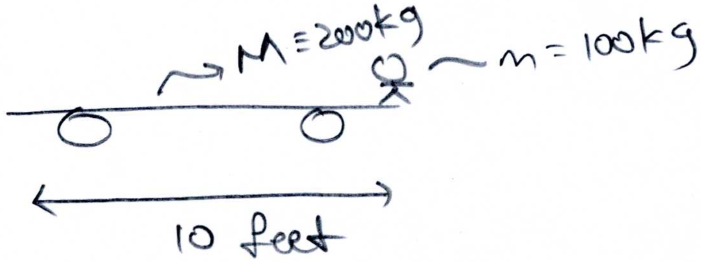
To find total com, we need to know the com contribution to region (2). However, it is just another triangle, but it is below the x-axis. By above, we get

$$y^{(2)}_{com} = -\frac{1}{3}b_2$$

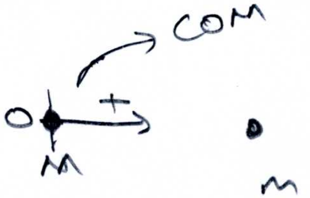
$$\Rightarrow \vec{y}_{com} = \vec{y}^{(1)}_{com} + \vec{y}^{(2)}_{com} = \frac{1}{3}(b_1 - b_2)\hat{j}$$

$$\Rightarrow \vec{r}_{com} = \left(0, \frac{1}{3}(b_1 - b_2), \frac{D}{2}\right)$$

3



(a) The system above is equivalent to:



(*note: coordinate system is chosen @ the COM of the cart)

$$X_{com} = \frac{M \cdot 0 + m \cdot 5}{M + m}$$

$$= \frac{100 \cdot 5}{300}$$

$$= \frac{5}{3} \text{ feet}$$

*again, this number is measured from our origin... so, X_{com} is located ≈ 1.67 feet to the right of the cart's COM.

(b)

There are no external forces on the system, so the X_{com} should be unchanged from a coordinate system on the ground. Since the person moving to the left changes the COM to the left, the cart will have to move to the right to keep the total COM unchanged. The person moves the COM by

$$\frac{5}{3} \cdot 2 = \frac{10}{3} \text{ feet},$$

So the cart will move to the right by this much.

(c) Again, no external forces in x-direction, so momentum is conserved:

coordinate system attached to ground

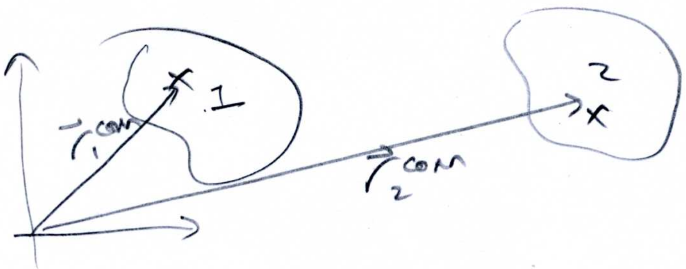
$$P_i = P_f$$

$$0 = M\vec{V} + m\vec{v}_0 \qquad \|\vec{v}_0\| = 1 \text{ m/s}$$

$$\Rightarrow \|\vec{V}\| = \frac{m}{M}\|\vec{v}_0\| = \frac{1}{2} \text{ m/s}$$

Since person moves left, $\vec{V}$ will point to the right.

Extra Credit!



$$\vec{r}^{\,com}_{system} = \frac{\int \vec{r}\,dm}{\int dm} = \frac{\int_1 \vec{r}\,dm + \int_2 \vec{r}\,dm}{\int_1 dm + \int_2 dm}$$

we also know: $\vec{r}_1^{\,com} = \dfrac{\int_1 \vec{r}\,dm}{\int_1 dM}$

$$\Rightarrow M_1 \vec{r}_1^{\,com} = \int_1 \vec{r}\,dm$$

I'm using $\int_1$ to denote the integral over all the object 1. This means $\int_1 dm = M_1$. Combining these results:

$$\vec{r}^{\,com}_{system} = \frac{M_1 \vec{r}_1^{\,com} + M_2 \vec{r}_2^{\,com}}{M_1 + M_2}$$

$$= \frac{\sum_i M_i \vec{r}_i^{\,com}}{\sum_i M_i}$$