4A - Quiz #3 Solutions - Winter 2016

I) $U(x) \equiv 2Ax^2 - 6Bx$, $A = 1\ J/m^2$
$B = 1\ J/m$

mass $\equiv m = 6$ kg.

$E = 20\ J$

(a) First, find the force, then we can find the acceleration:

$$\vec{F} = -\frac{dU}{dx}\hat{x} = -[4x - 6]\hat{x}$$

$$\Rightarrow \vec{F}(x=0) = +6\hat{x}$$

Now, we use $\vec{F} = m\vec{a}$

$$\Rightarrow +6\hat{x} = 6 \cdot \vec{a}$$

$$\Rightarrow a = +1\ m/s^2\ \hat{x}$$

(b) velocity can be found from KE:

$$E = KE + U \Rightarrow KE = E - U$$

$$\frac{1}{2}mv^2 = 20 - U(x=0)$$

$$\Rightarrow v = \sqrt{\frac{2}{m}[20-0]}$$

$$= \sqrt{\frac{40}{6}}$$

$$\approx 2.6\ m/s$$

(c) Allowed range is restricted by allowable values for KE (i.e. it must always be positive or zero).

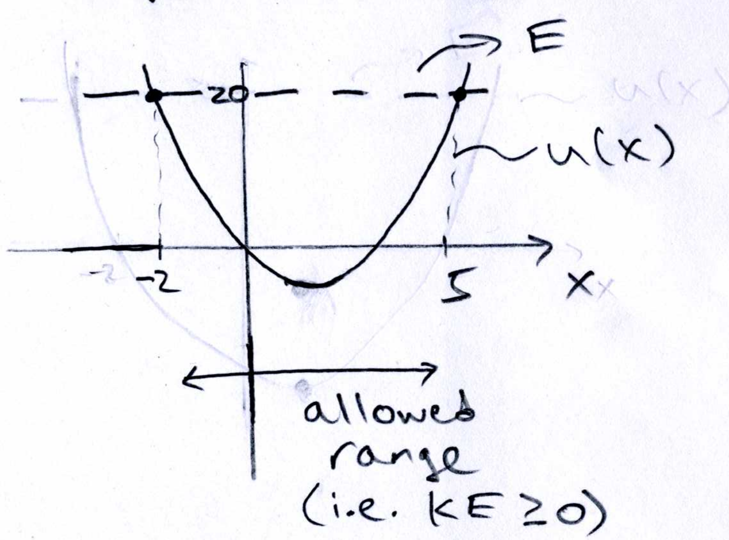
$$KE = 0 = E - U$$

$$\Rightarrow 20 = 2x^2 - 6x$$

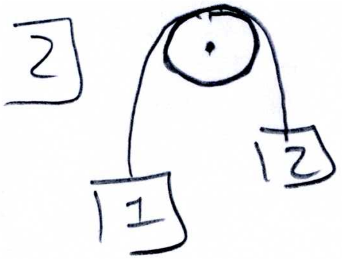
when $x = -2$ & $x = 5$, the $KE = 0$. Any x-values outside of this range gives a negative KE, which is unphysical.

$\Rightarrow$ range $= -2 \le x \le 5$

(d)



$\boxed{2}$



$m_1 > m_2$

(a) To find a, draw FBD & solve $F = ma$:

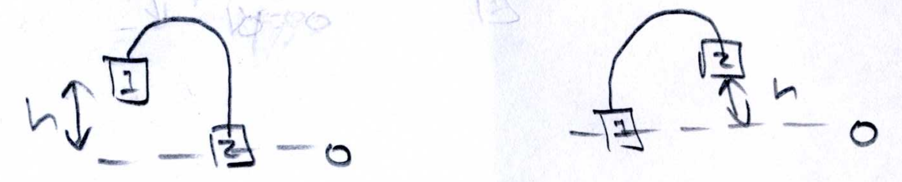
$F_{g1} - T = m_1 a$ & $T - F_{g2} = m_2 a$

$$\Rightarrow F_{g1} - F_{g2} = (m_1 + m_2) a$$

$$a = \frac{(m_1 - m_2) g}{m_1 + m_2}$$

(b) These masses exhibit linear motion since the displacement is the same as the trajectory.

(c) without loss of generality, lets put mass $\boxed{2}$ at a height of 0 initially:



$E_i = m_1 g h$ $E_f = m_2 g h + \frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2$

$$\Rightarrow \frac{1}{2}(m_1 v^2 + m_2 v^2) = m_1 g h - m_2 g h$$

$$\rightarrow v = \sqrt{2gh \left(\frac{m_1 - m_2}{m_1 + m_2}\right)}$$

* You could have also started out using the work energy theorem.

(d) $T - F_{g2} = m_2 a$

$$\Rightarrow T = F_{g2} + m_2 a$$
$$= m_2 g + m_2 a$$
$$= m_2 g \left[1 + \frac{m_1 - m_2}{m_1 + m_2}\right]$$

(e)

	$m_1 = 0$	$m_2 = 0$	$m_1 = m_2$
a	$-g$	$+g$	0
$\lVert\vec{v}\rVert$	$\sqrt{2gh}$	$\sqrt{2gh}$	0

All these results agree w/ our intuition.

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$m = 0.5$ kg
$\Delta x = 0.5$ m
$\theta = 30°$
$k = 10{,}000$ N/m

(a) $E_i = \frac{1}{2} k (\Delta x)^2$

$E_f = \frac{1}{2} m v^2$ ~ immediately after launch PE due to gravity is negligible.

$\frac{1}{2} k (\Delta x)^2 = \frac{1}{2} m v^2$

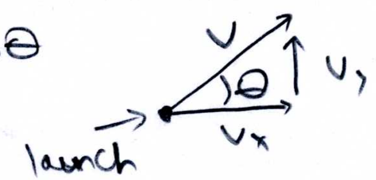
$\Rightarrow v = \Delta x \sqrt{\frac{k}{m}}$

$= 0.5 \sqrt{\frac{10{,}000}{0.5}}$

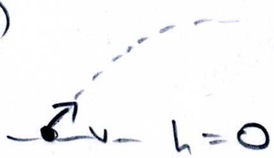
≈ 70.7 m/s

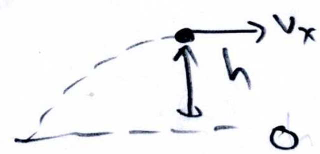
(b) @ its max height, the ball only has velocity in the x-direction:

$v_x = v \cos\theta$



(c)





$E_i = \frac{1}{2} m v^2$

$E_f = \frac{1}{2} m (v_x)^2 + mgh$

$E_i = E_f \Rightarrow h = \frac{v^2}{2g} (1 - \cos^2\theta)$

≈ 63.8 m

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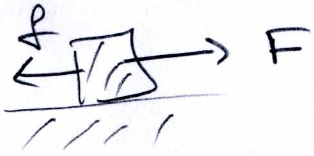
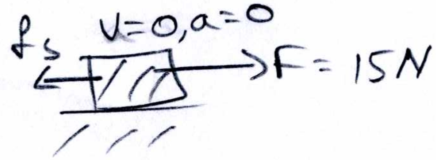
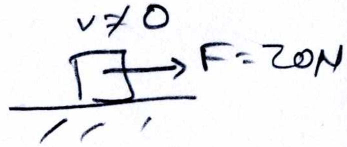


(a) 



the max. value of f_s is given by:

$$\mu_s mg$$

In the first case drawn above, we determine

$$\mu_s mg \geq 15$$

In the 2nd case, we have:

$$\mu_s mg < 20$$

$$\Rightarrow \frac{15}{mg} \leq \mu_s < \frac{20}{mg}, \quad mg = 100N$$

$$0.15 \leq \mu_s < 0.2$$

(b) There are no dynamical restrictions on μ_K. In most cases however, $\mu_K \leq \mu_S$.

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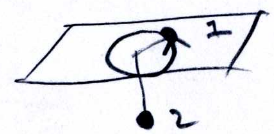
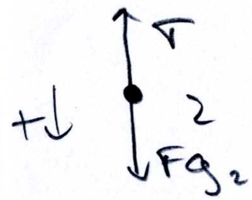


Draw FBD & Solve F=ma for either mass:



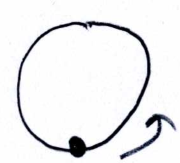
$$\Rightarrow Fg_2 - T = ma = 0$$

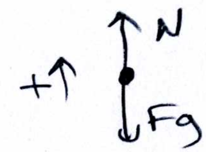
$$\Rightarrow T = m_2 g$$

Alternatively, we can look @ m_1:

$$T = m_1 a_c = \frac{m_1 v^2}{R}$$

6 Draw FBD & Solve F=ma:





(1) $N - F_g = ma_c$

(2) $N = 3F_g$

$$\Rightarrow v^2 = \frac{R}{m}[N - F_g]$$

$$= \frac{R}{m} 2F_g$$

$$= 2Rg$$

$$\Rightarrow v = \sqrt{2Rg}$$