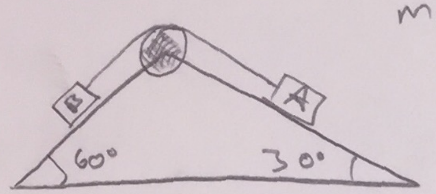
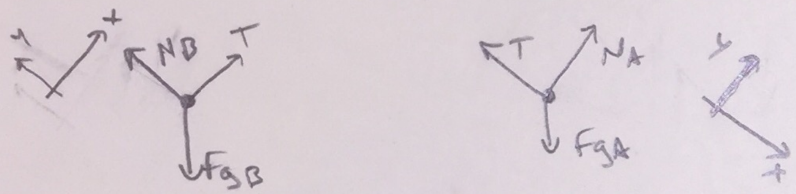
4A Quiz #2 Solutions - Winter 2016

1)

$m_A = 5\,kg$



(a) To solve, let's draw free body diagrams & choose coordinate systems for each mass:



x: $T - F_{gB,x} = m_B a_{B,x}$ $\qquad F_{gA,x} - T = m_A a_{A,x}$

y: $N_B - F_{gB,y} = m_B a_{B,y}$ $\qquad N_A - F_{gA,y} = m_A a_{A,y}$

$a_{B,x} = a_{B,y} = a_{A,x} = a_{A,y} = 0$

$F_{gB,x} = m_B g \sin(60)$ $\qquad F_{gA,x} = m_A g \sin(30)$

$F_{gB,y} = m_B g \cos(60)$ $\qquad F_{gA,y} = m_A g \cos(30)$

$\sin(60) = \cos(30) = \frac{\sqrt{3}}{2}$

$\sin(30) = \cos(60) = \frac{1}{2}$

using the equations in the x-direction:

$T = m_B g \frac{\sqrt{3}}{2}$ & $T = m_A g \left(\frac{1}{2}\right)$

$\Rightarrow m_B = \frac{1}{\sqrt{3}} m_A \approx 2.9\ kg$

(b) using the equations in the y-direction:

$N_B = m_B g \left(\frac{1}{2}\right)$ & $N_A = m_A g \left(\frac{\sqrt{3}}{2}\right)$

$\Rightarrow \frac{N_A}{N_B} = \frac{m_A g \left(\frac{\sqrt{3}}{2}\right)}{\frac{1}{\sqrt{3}} m_A g \left(\frac{1}{2}\right)} = 3$

(c) $m_B = 2 m_A = 10\,kg.$

$a_{B,y} = a_{A,y} = 0$

$a_{B,x} = a_{A,x} \equiv a$

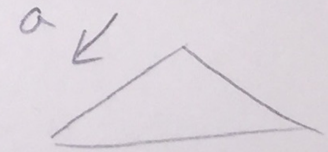
From the x-direction:

$F_{gA,x} - F_{gB,x} = (m_A + m_B) a$

$m_A g \left(\frac{1}{2}\right) - m_B g \left(\frac{\sqrt{3}}{2}\right) = (m_A + m_B) a$

$\Rightarrow a = \frac{\frac{g}{2}(m_A - \sqrt{3}\, m_B)}{m_A + m_B}$

$= -4.0\ m/s^2$



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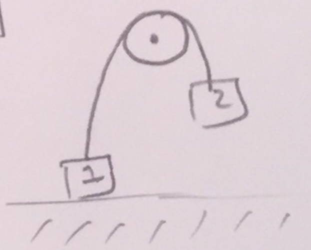
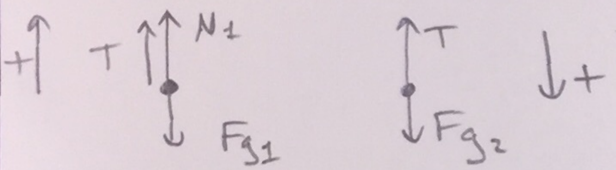


Again, first steps are to draw force diagram & choose coord. system then write down $F = ma$:



$T + N_1 - F_{g1} = m_1 a_1$ & $F_{g2} - T = m_2 a_2$

(a) $a_1 = a_2 = 0$

$\Rightarrow N_1 - F_{g1} + F_{g2} = 0$ ~ substitute in T

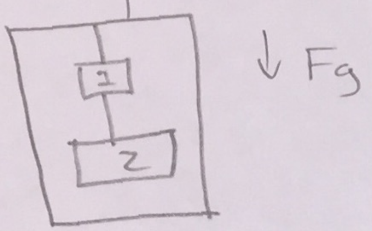
$N_1 = g(m_1 - m_2)$

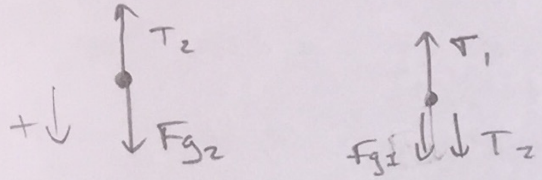
(b) $T_1 = F_{g1} - N_1$

(c) $T_2 = F_{g2}$

(d) $T_1 = T_2$ if the string is massless & if the pulley does not exert any additional forces on the string.

3)





(a) $F_{g2} - T_2 = -m_2 a$ & $F_{g1} + T_2 - T_1 = -m_1 a$

$\Rightarrow F_{g2} + F_{g1} - T_1 = -(m_1+m_2)a$

$T_1 = (m_1+m_2)a + (m_1+m_2)g$

(b) $T_2 = F_{g2} + m_2 a = m_2(g+a)$

(c) $F_{g2} - T_2 = m_2 a$ & $F_{g1} + T_2 - T_1 = m_1 a$

$\Rightarrow T_1 = (m_1+m_2)g - (m_1+m_2)a$

& $T_2 = m_2(g-a)$

The point @ which strings loose "tautness" is when $T \to 0$.

$0 = (m_1+m_2)g - (m_1+m_2)a$

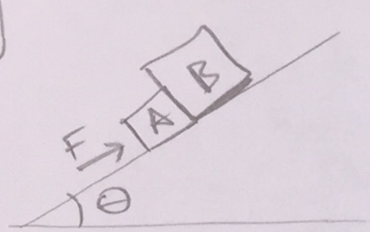
$\Rightarrow a = g$

$0 = m_2(g-a)$

$\Rightarrow a = g$

(d) Neither. They both become slack for the same acceleration.

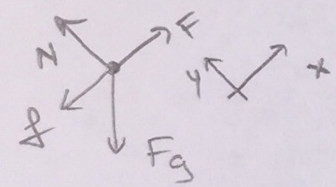
4)



given: μ_s, μ_k, θ, F
m_A, m_B

For parts (a) → (d) we can treat the two masses as one mass since that will be the system we apply Newton's laws to. Set $m = m_A + m_B$

(a)



$x: \; F - mg\sin\theta - f = ma_x = 0 \sim$ not moving

$y: \; N - mg\cos\theta = ma_y = 0 \sim$ not moving

$\Rightarrow N = mg\cos\theta$

& $F = mg\sin\theta + f$

block's don't move for $f < \mu_s N$

$\Rightarrow F < mg(\sin\theta + \mu_s\cos\theta)$

(b) the point @ which blocks will move is around $f = \mu_s N = \mu_s mg\cos\theta$

$\Rightarrow F = mg(\sin\theta + \mu_s\cos\theta)$

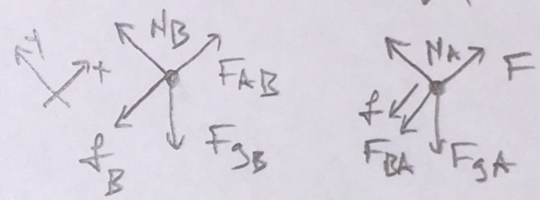
(c) The applied force must be large enough to overcome static friction for the system to start to move:

$$F > mg(\sin\theta + \mu_s\cos\theta)$$

(d) $F - mg\sin\theta - \mu_k N = ma$
$N - mg\cos\theta = 0$

$$\Rightarrow a = \frac{F - mg(\sin\theta + \mu_k\cos\theta)}{m}$$

(e)



$F_{AB} \equiv$ force A applies on B

$B: \; x: \; F_{AB} - f_B - m_B g\sin\theta = m_B a$

$y: \; N_B - m_B g\cos\theta = 0$

$f_B = \mu_k N_B = \mu_k m_B g\cos\theta$

$\Rightarrow F_{AB} = f_B + m_B g\sin\theta + m_B a$

$= m_B g(\sin\theta + \mu_k\cos\theta) + m_B\left[\frac{F - mg(\sin\theta + \mu_k\cos\theta)}{m}\right]$

$= \frac{m_B}{m}F$

$= \frac{m_B}{m_A + m_B}F$

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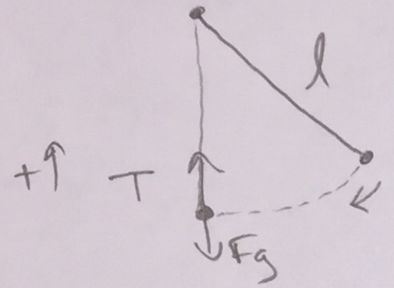


$$T - F_g = m a_c = m \frac{v^2}{\ell}$$

$$\Rightarrow T = w + m \frac{v^2}{\ell}, \qquad w \equiv F_g.$$

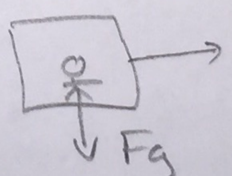
6

(a) For $t<0$, she is in an inertial reference frame because she is not accelerating.

(b) However, $t>0$, she starts to accelerate because there is a force on her. At this point, she is no longer in an inertial coordinate system.

(c) If the whole coord. system starts to accelerate, she will feel a force. For instance, if she is in a car ~~that~~ initially starts @ rest, but then picks up speed, she will feel a horizontal force from her chair due to its acceleration.

$$\text{weight} \equiv w = mg \Rightarrow m = \frac{1}{g} w$$



$$F = ma = \frac{a}{g} w$$

given an acceleration, the force she will feel is

$$F = \frac{a}{g} w$$