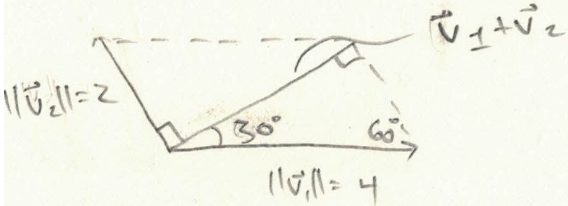
4A Quiz #1 Solutions - Winter 2016

1. $\|\vec{v}_1\| = 4$, $\|\vec{v}_2\| = 2$, $\theta_{deg} = 120°$

a) $\theta_{rad} = \theta_{deg} \cdot \frac{\pi}{180} = \frac{2}{3}\pi$

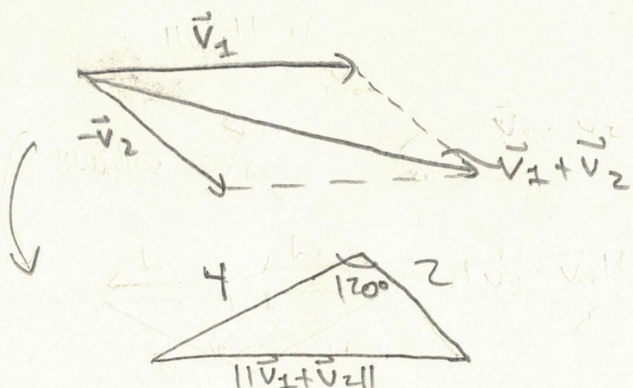
b) (First method)



$$\Rightarrow \cos(30°) = \frac{\|\vec{v}_1+\vec{v}_2\|}{4}$$

$$\rightarrow \|\vec{v}_1+\vec{v}_2\| = 4\frac{\sqrt{3}}{2} = 2\sqrt{3}$$

c) (First method)



using the law of cosines gives:

$$\|\vec{v}_1+\vec{v}_2\| = \sqrt{4^2+2^2-2(4)(2)\cos(120°)}$$
$$= \sqrt{16+4-2\cdot 4\cdot 2\cdot -\tfrac{1}{2}}$$

b) (Second method)

$$\|\vec{v}_1+\vec{v}_2\|^2 = (\vec{v}_1+\vec{v}_2)\cdot(\vec{v}_1+\vec{v}_2) \sim \text{dot product}$$
$$= \|\vec{v}_1\|^2 + \|\vec{v}_2\|^2 + 2\vec{v}_1\cdot\vec{v}_2$$
$$= 4^2 + 2^2 + 2\vec{v}_1\cdot\vec{v}_2$$

$$\vec{v}_1\cdot\vec{v}_2 = \|\vec{v}_1\|\,\|\vec{v}_2\|\cos\left(\frac{2\pi}{3}\right)$$
$$= -4$$

$$\Rightarrow \|\vec{v}_1+\vec{v}_2\| = \sqrt{16+4+2(-4)} = 2\sqrt{3}$$

c) (Second method)

$$\|\vec{v}_1-\vec{v}_2\| = \sqrt{\|\vec{v}_1\|^2+\|\vec{v}_2\|^2-2\vec{v}_1\cdot\vec{v}_2}$$
$$= 2\sqrt{7}$$

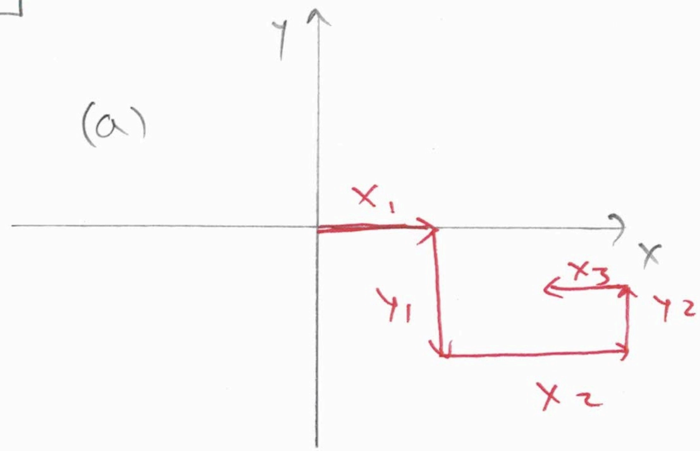
d) $\vec{v}_1\cdot\vec{v}_2 = -4$

* Note: Sometimes "$\cdot$" represents the dot product & sometimes it represents normal multiplication. Should be clear from the context what it is.

* Note: all angles in the first method can be filled in using normal geometry/trig arguments.

2

(a)



(b)

$$\left.\begin{aligned} \vec{r}_1 &\equiv (x_1, -y_1) \\ \vec{r}_2 &\equiv (x_2, y_2) \\ \vec{r}_3 &\equiv (-x_3, 0) \end{aligned}\right\} \rightarrow \Delta\vec{r} = (\vec{r}_1 + \vec{r}_2 + \vec{r}_3) - \vec{o} = (x_1 + x_2 - x_3,\ y_2 - y_1)$$

$$\vec{o} \equiv (0,0)$$

If we also wanted to know the magnitude of displacement, we would do:

$$\|\Delta\vec{r}\| = \sqrt{(x_1 + x_2 - x_3)^2 + (y_2 - y_1)^2}$$

c) $\text{dist} = x_1 + x_2 + x_3 + y_1 + y_2$

d) $T = \dfrac{\text{dist}}{v} = \dfrac{x_1 + x_2 + x_3 + y_1 + y_2}{v}$

3) $x = \frac{1}{2}aT^2 + V_oT + x_o$

$V_o = 6\,m/s \rightarrow V_f = 25\,m/s^2. \quad a = 3\,m/s^2$

we also know that

$V_f = V_o + aT \Rightarrow T = \dfrac{V_f - V_o}{a}$

Plugging in T to the first expression gives:

$$\Delta x = \frac{1}{2}a\left(\frac{V_f - V_o}{a}\right)^2 + V_o\left(\frac{V_f - V_o}{a}\right)$$

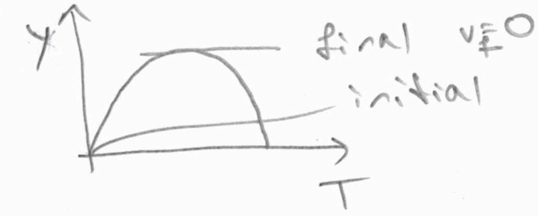
$$= \frac{1}{2}(3)\left(\frac{25-6}{3}\right)^2 + 6\left(\frac{25-6}{3}\right)$$

$$\approx 98.2\,m$$

* Note:

unknowns = 2 (i.e. x & T)

equation = 2

4) 

$g = +9.8\,m/s^2$

a) $y = \frac{1}{2}aT^2 + V_oT + y_o$

$y_o = 0 \quad \& \quad V_o = V$

Also, $T = \dfrac{V_f - V_o}{a} = \dfrac{0 - V}{-g} = \dfrac{V}{g}$

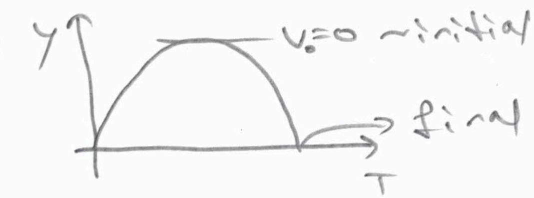
So, $y = \frac{1}{2}(-g)\dfrac{V^2}{g^2} + V\left(\dfrac{V}{g}\right)$

$\Rightarrow y_{max} = \dfrac{V^2}{2g}$

* Check to make sure these units work:

$m = \left(\frac{m}{s}\right)^2 \div \frac{m}{s^2} = \frac{m^2}{s^2} \cdot \frac{s^2}{m} = m$

b) Solved above

c) 

$y = \frac{1}{2}aT^2 + V_oT + y_o \quad \& \quad T = \dfrac{V_f - V_o}{a}$

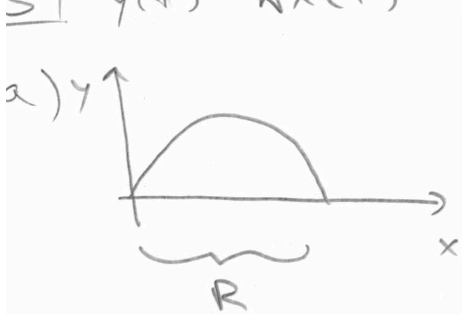
$0 = \frac{1}{2}(-g)\left(\dfrac{V_f - 0}{-g}\right)^2 + y_{max}$

$-\dfrac{V^2}{2g} = -\dfrac{V_f^2}{2g}$

$\Rightarrow V_f = V$

* Since we ignore friction, the ball comes back down @ the same speed we launched V with

5) $y(t) = Ax(t)^2 + Bx(t)$, $A<0$ & $B>0$

a) 

To find R, we set $y(t)=0$ & solve for x:

$0 = Ax^2 + Bx \Rightarrow x=0$ or $x = -\frac{B}{A}$

Since $A<0$, $x = \frac{-B}{A} > 0$ (i.e. it is a positive value).

So, $R = -\frac{B}{A}$.

b) set $\frac{dy}{dx} = 0$

$2Ax + B = 0 \Rightarrow x_{high} = -\frac{B}{2A}$

$y_{high} = A\left(-\frac{B}{2A}\right)^2 + B\left(-\frac{B}{2A}\right)$

$= -\frac{B^2}{4A}$

So, $(x_{high}, y_{high}) = \left(-\frac{B}{2A}, -\frac{B^2}{4A}\right)$

*notice how $x_{high} = \frac{R}{2}$, which is what we expect for parabolic motion.

(c,d,e) $y = Ax^2 + Bx \rightarrow \frac{dy}{dx} = 2Ax + B$

$\frac{dy}{dx}$ @ origin $= 2A(0) + B = B$

$\frac{dy}{dx}$ @ $x=R$ $= 2A(R) + B = -B$

we also know that $\frac{dy}{dx} = \tan\theta$

$\Rightarrow \tan\theta_0 = B$ @ origin

$\theta_0 = \tan^{-1}(B)$

& $\tan\theta_f = -B$ @ $x=R$

$\theta_f = -\tan^{-1}(B) = -\theta_0$

*Notice how $\theta_f = -\theta_0$, which is what we expect since the problem is symmetric ... if the ball initially is launched 30° above the x-axis, it will strike the ground w/ velocity pointed 30° below the x-axis.

6) $R=0$. The ball needs to be launched up some amount to move anywhere in our model. To compare w/ problem 5:

$R = -\frac{B}{A} = \frac{-\tan\theta}{A} = -\lim_{\epsilon\to 0} \frac{\tan\epsilon}{A} = 0$