PHYS 4A – Mechanics by Prof. Hans Paar – Winter 2016

Chapter 9 Solutions

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11)

Consider the motion in one dimension, with the positive direction being the direction of motion of the alpha particle. Let A represent the alpha particle, with a mass of m_A, and let B represent the daughter nucleus, with a mass of $57m_A$. The total momentum must be 0 since the nucleus decayed at rest.

$$p_{\text{initial}} = p_{\text{final}} \rightarrow 0 = m_A v'_A + m_B v'_B \rightarrow$$

$$v'_B = -\frac{m_A v'_A}{m_B} = -\frac{m_A\left(2.8\times10^5\ \text{m/s}\right)}{57m_A} \rightarrow \left|v'_B\right| = \boxed{4900\,\text{m/s}}$$

Note that the masses do not have to be converted to kg, since all masses are in the same units, and a ratio of masses is what is significant.

15)

Momentum will be conserved in one dimension in the explosion. Let A represent the fragment with the larger kinetic energy.

$$p_{\text{initial}} = p_{\text{final}} \rightarrow 0 = m_A v'_A + m_B v'_B \rightarrow v'_B = -\frac{m_A v'_A}{m_B}$$

$$K_A = 2K_B \rightarrow \tfrac{1}{2}m_A v'^2_A = 2\left(\tfrac{1}{2}m_B v'^2_B\right) = m_B\left(-\frac{m_A v'_A}{m_B}\right)^2 \rightarrow \frac{m_A}{m_B} = \boxed{\frac{1}{2}}$$

The fragment with the larger kinetic energy has half the mass of the other fragment.

19)

Since no outside force acts on the two masses, their total momentum is conserved.

$$m_1\vec{\mathbf{v}}_1 = m_1\vec{\mathbf{v}}'_1 + m_2\vec{\mathbf{v}}'_2 \rightarrow$$

$$\vec{\mathbf{v}}'_2 = \frac{m_1}{m_2}\left(\vec{\mathbf{v}}_1 - \vec{\mathbf{v}}'_1\right) = \frac{2.0\,\text{kg}}{3.0\,\text{kg}}\left[\left(4.0\hat{\mathbf{i}} + 5.0\hat{\mathbf{j}} - 2.0\hat{\mathbf{k}}\right)\text{m/s} - \left(-2.0\hat{\mathbf{i}} + 3.0\hat{\mathbf{k}}\right)\text{m/s}\right]$$

$$= \frac{2.0\,\text{kg}}{3.0\,\text{kg}}\left[\left(6.0\hat{\mathbf{i}} + 5.0\hat{\mathbf{j}} - 5.0\hat{\mathbf{k}}\right)\text{m/s}\right]$$

$$= \boxed{\left(4.0\hat{\mathbf{i}} + 3.3\hat{\mathbf{j}} - 3.3\hat{\mathbf{k}}\right)\text{m/s}}$$

25)

The impulse given the ball is the change in the ball's momentum. From the symmetry of the problem, the vertical momentum of the ball does not change, and so there is no vertical impulse. Call the direction AWAY from the wall the positive direction for momentum perpendicular to the wall.

$$\Delta p_{\perp} = mv_{\perp_{\text{final}}} - mv_{\perp_{\text{initial}}} = m\left(v\sin 45^{\circ} - -v\sin 45^{\circ}\right) = 2mv\sin 45^{\circ}$$

$$= 2\left(6.0\times10^{-2}\,\text{km}\right)\left(25\,\text{m/s}\right)\sin 45^{\circ} = \boxed{2.1\,\text{kg}\bullet\text{m/s, to the left}}$$

36)

(*a*) Momentum will be conserved in one dimension. Call the direction of the first ball the positive direction. Let A represent the first ball, and B represent the second ball. We have $v_B = 0$ and $v'_B = \frac{1}{2}v_A$. Use Eq. 9-8 to obtain a relationship between the velocities.

$$v_A - v_B = -\left(v'_A - v'_B\right) \;\rightarrow\; v'_A = -\tfrac{1}{2}v_A$$

Substitute this relationship into the momentum conservation equation for the collision.

$$p_{\text{initial}} = p_{\text{final}} \;\rightarrow\; m_A v_A + m_B v_B = m_A v'_A + m_B v'_B \;\rightarrow\; m_A v_A = -\tfrac{1}{2}m_A v_A + m_B \tfrac{1}{2}v_A \;\rightarrow$$

$$m_B = 3m_A = 3\left(0.280\text{ kg}\right) = \boxed{0.840\text{ kg}}$$

(*b*) The fraction of the kinetic energy given to the second ball is as follows.

$$\frac{K'_B}{K_A} = \frac{\frac{1}{2}m_B v_B'^2}{\frac{1}{2}m_A v_A^2} = \frac{3m_A\left(\frac{1}{2}v_A\right)^2}{m_A v_A^2} = \boxed{0.75}$$

40)

Both momentum and kinetic energy are conserved in this one-dimensional collision. We start with Eq. 9-3 (for a one-dimensional setting) and Eq. 9-8.

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B \;;\; v_A - v_B = -\left(v'_A - v'_B\right) \;\rightarrow\; v'_B = v_A - v_B + v'_A$$

Insert the last result above back into the momentum conservation equation.

$$m_A v_A + m_B v_B = m_A v'_A + m_B\left(v_A - v_B + v'_A\right) = \left(m_A + m_B\right)v'_A + m_B\left(v_A - v_B\right) \;\rightarrow$$

$$m_A v_A + m_B v_B - m_B\left(v_A - v_B\right) = \left(m_A + m_B\right)v'_A \;\rightarrow\; \left(m_A - m_B\right)v_A + 2m_B v_B = \left(m_A + m_B\right)v'_A \;\rightarrow$$

$$\boxed{v'_A = v_A\left(\frac{m_A - m_B}{m_A + m_B}\right) + v_B\left(\frac{2m_B}{m_A + m_B}\right)}$$

Do a similar derivation by solving Eq. 9-8 for v'_A, which gives $v'_A = v'_B - v_A + v_B$.

$$m_A v_A + m_B v_B = m_A\left(v'_B - v_A + v_B\right) + m_B v'_B = m_A\left(-v_A + v_B\right) + \left(m_A + m_B\right)v'_B \;\rightarrow$$

$$m_A v_A + m_B v_B - m_A\left(-v_A + v_B\right) = \left(m_A + m_B\right)v'_B \;\rightarrow\; 2m_A v_A + \left(m_B - m_A\right)v_B = \left(m_A + m_B\right)v'_B \;\rightarrow$$

$$\boxed{v'_B = v_A\left(\frac{2m_A}{m_A + m_B}\right) + v_B\left(\frac{m_B - m_A}{m_A + m_B}\right)}$$

43)

(*a*) In Example 9-11, $K_i = \frac{1}{2}mv^2$ and $K_f = \frac{1}{2}(m+M)v'^2$. The speeds are related by

$$v' = \frac{m}{m+M}v.$$

$$\frac{\Delta K}{K_i} = \frac{K_f - K_i}{K_i} = \frac{\frac{1}{2}(m+M)v'^2 - \frac{1}{2}mv^2}{\frac{1}{2}mv^2} = \frac{(m+M)\left(\frac{m}{m+M}v\right)^2 - mv^2}{mv^2}$$

$$= \frac{\frac{m^2v^2}{m+M} - mv^2}{mv^2} = \frac{m}{m+M} - 1 = \boxed{\frac{-M}{m+M}}$$

(*b*) For the given values, $\frac{-M}{m+M} = \frac{-380\text{ g}}{396\text{ g}} = \boxed{-0.96}$. Thus 96% of the energy is lost.

46)

Use conservation of momentum in one dimension. Call the direction of the sports car's velocity the positive x direction. Let A represent the sports car, and B represent the SUV. We have $v_B = 0$ and $v'_A = v'_B$. Solve for v_A.

$$p_{\text{initial}} = p_{\text{final}} \rightarrow m_A v_A + 0 = (m_A + m_B)v'_A \rightarrow v_A = \frac{m_A + m_B}{m_A}v'_A$$

The kinetic energy that the cars have immediately after the collision is lost due to negative work done by friction. The work done by friction can also be calculated using the definition of work. We assume the cars are on a level surface, so that the normal force is equal to the weight. The distance the cars slide forward is Δx. Equate the two expressions for the work done by friction, solve for v'_A, and use that to find v_A.

$$W_{\text{fr}} = (K_{\text{final}} - K_{\text{initial}})_{\substack{\text{after}\\ \text{collision}}} = 0 - \frac{1}{2}(m_A + m_B)v'^2_A$$

$$W_{\text{fr}} = F_{\text{fr}}\Delta x\cos 180^\circ = -\mu_k(m_A + m_B)g\Delta x$$

$$-\frac{1}{2}(m_A + m_B)v'^2_A = -\mu_k(m_A + m_B)g\Delta x \rightarrow v'_A = \sqrt{2\mu_k g\Delta x}$$

$$v_A = \frac{m_A + m_B}{m_A}v'_A = \frac{m_A + m_B}{m_A}\sqrt{2\mu_k g\Delta x} = \frac{920\,\text{kg} + 2300\,\text{kg}}{920\,\text{kg}}\sqrt{2(0.80)(9.8\,\text{m/s}^2)(2.8\,\text{m})}$$

$$= 23.191\,\text{m/s} \approx \boxed{23\,\text{m/s}}$$

48)

$$\text{Fraction } K \text{ lost} = \frac{K_{\text{initial}} - K_{\text{final}}}{K_{\text{initial}}} = \frac{\frac{1}{2}m_A v_A^2 - \frac{1}{2}m_B v'^2_B}{\frac{1}{2}m_A v_A^2} = \frac{v_A^2 - v'^2_B}{v_A^2} = \frac{(35\,\text{m/s})^2 - (25\,\text{m/s})^2}{(35\,\text{m/s})^2} = \boxed{0.49}$$

50)

The swinging motion will conserve mechanical energy. Take the zero level for gravitational potential energy to be at the bottom of the arc. For the pendulum to swing exactly to the top of the arc, the potential energy at the top of the arc must be equal to the kinetic energy at the bottom.

$$K_{\text{bottom}} = U_{\text{top}} \rightarrow \tfrac{1}{2}(m+M)V_{\text{bottom}}^2 = (m+M)g(2L) \rightarrow V_{\text{bottom}} = 2\sqrt{gL}$$

Momentum will be conserved in the totally inelastic collision at the bottom of the arc. We assume that the pendulum does not move during the collision process.

$$p_{\text{initial}} = p_{\text{final}} \rightarrow mv = (m+M)V_{\text{bottom}} \rightarrow v = \frac{m+M}{m} = \boxed{2\frac{m+M}{m}\sqrt{gL}}$$

64)

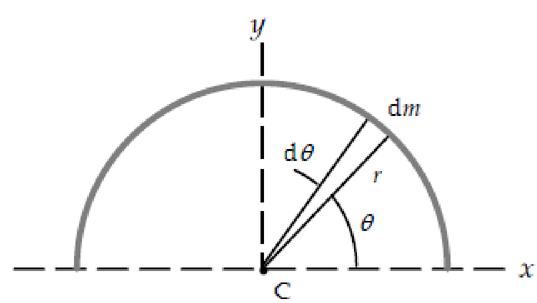
By the symmetry of the problem, since the centers of the cubes are along a straight line, the vertical CM coordinate will be 0, and the depth CM coordinate will be 0. The only CM coordinate to calculate is the one along the straight line joining the centers. The mass of each cube will be the volume times the density, and so $m_1 = \rho(\ell_0)^3$, $m_2 = \rho(2\ell_0)^3$, $m_3 = \rho(3\ell_0)^3$. Measuring from the left edge of the smallest block, the locations of the CMs of the individual cubes are $x_1 = \frac{1}{2}\ell_0$, $x_2 = 2\ell_0$, $x_3 = 4.5\ell_0$. Use Eq. 9-10 to calculate the CM of the system.

$$x_{\text{CM}} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{\rho\ell_0^3(\frac{1}{2}\ell_0) + 8\rho\ell_0^3(2\ell_0) + 27\rho\ell_0^3(4.5\ell_0)}{\rho\ell_0^3 + 8\rho\ell_0^3 + 27\rho\ell_0^3}$$

$$= \boxed{3.8\ell_0 \text{ from the left edge of the smallest cube}}$$

67)

From the symmetry of the wire, we know that $x_{\text{CM}} = 0$. Consider an infinitesimal piece of the wire, with mass dm, and coordinates $(x, y) = (r\cos\theta, r\sin\theta)$. If the length of that piece of wire is $d\ell$, then since the wire is uniform, we have $dm = \frac{M}{\pi r}d\ell$. And from the diagram and the definition of radian angle measure, we have $d\ell = rd\theta$. Thus $dm = \frac{M}{\pi r}rd\theta = \frac{M}{\pi}d\theta$. Now apply Eq. 9-13.



$$y_{\text{CM}} = \frac{1}{M}\int y\,dm = \frac{1}{M}\int_0^{\pi} r\sin\theta\frac{M}{\pi}d\theta = \frac{r}{\pi}\int_0^{\pi}\sin\theta d\theta = \frac{2r}{\pi}$$

Thus the coordinates of the center of mass are $(x_{\text{CM}}, y_{\text{CM}}) = \boxed{\left(0, \frac{2r}{\pi}\right)}$.

94)

(*a*) No, there is no net external force on the system. In particular, the spring force is internal to the system.

(*b*) Use conservation of momentum to determine the ratio of speeds. Note that the two masses will be moving in opposite directions. The initial momentum, when the masses are released, is 0.

$$p_{\text{initial}} = p_{\text{later}} \rightarrow 0 = m_A v_A - m_B v_B \rightarrow v_A/v_B = \boxed{m_B/m_A}$$

(*c*) $$\frac{K_A}{K_B} = \frac{\frac{1}{2}m_A v_A^2}{\frac{1}{2}m_B v_B^2} = \frac{m_A}{m_B}\left(\frac{v_A}{v_B}\right)^2 = \frac{m_A}{m_B}\left(\frac{m_B}{m_A}\right)^2 = \boxed{m_B/m_A}$$

(*d*) The center of mass was initially at rest. Since there is no net external force on the system, the center of mass does not move, and so stays at rest.

(*e*) With friction present, there could be a net external force on the system, because the forces of friction on the two masses would not necessarily be equal in magnitude. If the two friction forces are not equal in magnitude, the ratios found above would not be valid. Likewise, the center of mass would not necessarily be at rest with friction present.

98)

This is a ballistic "pendulum" of sorts, similar to Example 9-11 in the textbook. The mass of the bullet is m, and the mass of the block of wood is M. The speed of the bullet before the collision is v, and the speed of the combination after the collision is v'. Momentum is conserved in the totally inelastic collision, and so $mv = (m+M)v'$. The kinetic energy present immediately after the collision is lost due to negative work being done by friction.

$$W_{\text{fr}} = \Delta K = \tfrac{1}{2}m\left(v_f^2 - v_i^2\right)_{\substack{\text{after}\\\text{collision}}} \qquad W_{\text{fr}} = F_{\text{fr}}\Delta x \cos 180^\circ \theta = -\mu_k F_N \Delta x = -\mu_k mg\Delta x \rightarrow$$

$$-\mu_k g\Delta x = \tfrac{1}{2}\left(v_f^2 - v_i^2\right) = -\tfrac{1}{2}v'^2 \rightarrow v' = \sqrt{2\mu_k g\Delta x}$$

Use this expression for v' in the momentum conservation equation in one dimension in order to solve for v.

$$mv = (m+M)v' = (m+M)\sqrt{2\mu_k g\Delta x} \rightarrow$$

$$v = \left(\frac{m+M}{m}\right)\sqrt{2\mu_k g\Delta x} = \left(\frac{0.022\text{ kg} + 1.35\text{ kg}}{0.022\text{ kg}}\right)\sqrt{2(0.28)(9.80\text{ m/s}^2)(8.5\text{ m})}$$

$$= \boxed{4.3\times10^2\text{ m/s}}$$

111)

We apply Eq. 9-19b, with no external forces. We also assume that the motion is all in one dimension.

$$M\frac{d\bar{\mathbf{v}}}{dt} = \bar{\mathbf{v}}_{\text{rel}}\frac{dM}{dt} \rightarrow Mdv = v_{\text{rel}}dM \rightarrow \frac{1}{v_{\text{rel}}}dv = \frac{1}{M}dM \rightarrow$$

$$\frac{1}{v_{\text{rel}}}\int_0^{v_{\text{final}}} dv = \int_{M_0}^{M_{\text{final}}}\frac{1}{M}dM \rightarrow \frac{v_{\text{final}}}{v_{\text{rel}}} = \ln\frac{M_{\text{final}}}{M_0} \rightarrow M_{\text{final}} = M_0 e^{v_{\text{final}}/v_{\text{rel}}} \rightarrow$$

$$M_{\text{ejected}} = M_0 - M_{\text{final}} = M_0\left(1 - e^{v_{\text{final}}/v_{\text{rel}}}\right) = (210\text{kg})\left(1 - e^{2.0/(-35)}\right) = 11.66\text{kg} \approx \boxed{12\text{kg}}$$