PHYS 4A – Mechanics by Prof. Hans Paar – Winter 2016

Chapter 8 Solutions

Prepared by: Thyagarajan V

10)

Use Eq. 8-6 to find the potential energy function.

$$U(x) = -\int F(x)\,dx + C = -\int A\sin(kx)\,dx + C = \frac{A}{k}\cos(kx) + C$$

$$U(0) = \frac{A}{k} + C = 0 \quad\rightarrow\quad C = -\frac{A}{k} \quad\rightarrow\quad \boxed{U(x) = \frac{A}{k}\left[\cos(kx) - 1\right]}$$

23)

At the release point the mass has both kinetic energy and elastic potential energy. The total energy is $\frac{1}{2}mv_0^2 + \frac{1}{2}kx_0^2$. If friction is to be ignored, then that total energy is constant.

(*a*) The mass has its maximum speed at a displacement of 0, and so only has kinetic energy at that point.

$$\tfrac{1}{2}mv_0^2 + \tfrac{1}{2}kx_0^2 = \tfrac{1}{2}mv_{\max}^2 \quad\rightarrow\quad v_{\max} = \boxed{\sqrt{v_0^2 + \frac{k}{m}x_0^2}}$$

(*b*) The mass has a speed of 0 at its maximum stretch from equilibrium, and so only has potential energy at that point.

$$\tfrac{1}{2}mv_0^2 + \tfrac{1}{2}kx_0^2 = \tfrac{1}{2}kx_{\max}^2 \quad\rightarrow\quad x_{\max} = \boxed{\sqrt{x_0^2 + \frac{m}{k}v_0^2}}$$

26)

The maximum acceleration of 5.0 *g* occurs where the force is at a maximum. The maximum force occurs at the maximum displacement from the equilibrium of the spring. The acceleration and the displacement are related by Newton's second law and the spring law, $F_{\text{net}} = F_{\text{spring}} \rightarrow ma = -kx$ $\rightarrow x = -\frac{m}{k}a$. Also, by conservation of energy, the initial kinetic energy of the car will become the final potential energy stored in the spring.

$$E_{\text{initial}} = E_{\text{final}} \quad\rightarrow\quad \tfrac{1}{2}mv_0^2 = \tfrac{1}{2}kx_{\max}^2 = \tfrac{1}{2}k\left(\frac{m}{k}a_{\max}\right)^2 = \tfrac{1}{2}\frac{m^2}{k}(5.0g)^2 \quad\rightarrow$$

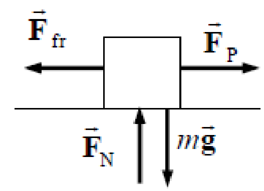
$$k = \frac{m(5.0g)^2}{v_0^2} = \frac{(1200\,\text{kg})\,25\left(9.80\,\text{m/s}^2\right)^2}{\left[95\,\text{km/h}\left(\dfrac{1.0\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2} = \boxed{4100\,\text{N/m}}$$

29)

Use conservation of energy, where all of the kinetic energy is transformed to thermal energy.

$$E_{\text{initial}} = E_{\text{final}} \rightarrow \tfrac{1}{2}mv^2 = E_{\text{thermal}} = \tfrac{1}{2}(2)(56{,}000\,\text{kg})\left[(95\,\text{km/h})\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2 = \boxed{3.9\times10^7\,\text{J}}$$

33)



We apply the work-energy theorem. There is no need to use potential energy since the crate moves along the level floor, and there are no springs in the problem. There are two forces doing work in this problem – the pulling force and friction. The starting speed is $v_0 = 0$. Note that the two forces do work over different distances.

$$W_{\text{net}} = W_{\text{P}} + W_{\text{fr}} = F_{\text{P}}d_{\text{P}}\cos 0^\circ + F_{\text{fr}}d_{\text{fr}}\cos 180^\circ = \Delta K = \tfrac{1}{2}m\left(v_f^2 - v_i^2\right) \rightarrow$$

$$F_{\text{P}}d_{\text{P}} - \mu_k mgd_{\text{fr}} = \tfrac{1}{2}mv_f^2 \rightarrow v_f = \sqrt{\frac{2}{m}\left(F_{\text{P}}d_{\text{P}} - \mu_k mgd_{\text{fr}}\right)}$$

$$= \sqrt{\frac{2}{(96\,\text{kg})}\left[(350\,\text{N})(30\,\text{m}) - (0.25)(96\,\text{kg})\left(9.80\,\text{m/s}^2\right)(15\,\text{m})\right]} = \boxed{12\,\text{m/s}}$$

39)

(*a*) Calculate the energy of the ball at the two maximum heights, and subtract to find the amount of energy lost. The energy at the two heights is all gravitational potential energy, since the ball has no kinetic energy at those maximum heights.

$$E_{\text{lost}} = E_{\text{initial}} - E_{\text{final}} = mgy_{\text{initial}} - mgy_{\text{final}}$$

$$\frac{E_{\text{lost}}}{E_{\text{initial}}} = \frac{mgy_{\text{initial}} - mgy_{\text{final}}}{mgy_{\text{initial}}} = \frac{y_{\text{initial}} - y_{\text{final}}}{y_{\text{initial}}} = \frac{2.0\text{ m} - 1.5\text{ m}}{2.0\text{ m}} = 0.25 = \boxed{25\%}$$

(*b*) The ball's speed just before the bounce is found from the initial gravitational potential energy, and the ball's speed just after the bounce is found from the ball's final gravitational potential energy.

$$U_{\text{initial}} = K_{\text{before}} \rightarrow mgy_{\text{initial}} = \tfrac{1}{2}mv_{\text{before}}^2 \rightarrow$$

$$v_{\text{before}} = \sqrt{2gy_{\text{initial}}} = \sqrt{2\left(9.80\,\text{m/s}^2\right)(2.0\,\text{m})} = \boxed{6.3\,\text{m/s}}$$

$$U_{\text{final}} = K_{\text{after}} \rightarrow mgy_{\text{final}} = \tfrac{1}{2}mv_{\text{after}}^2 \rightarrow$$

$$v_{\text{after}} = \sqrt{2gy_{\text{final}}} = \sqrt{2\left(9.80\,\text{m/s}^2\right)(1.5\text{ m})} = \boxed{5.4\,\text{m/s}}$$

(*c*) The energy "lost" was changed $\boxed{\text{primarily into heat energy}}$ – the temperature of the ball and the ground would have increased slightly after the bounce. Some of the energy may have been changed into acoustic energy (sound waves). Some may have been lost due to non-elastic deformation of the ball or ground.

42)

(*a*) Use conservation of energy. Subscript 1 represents the block at the compressed location, and subscript 2 represents the block at its maximum position up the slope. The initial location of the block at the bottom of the plane is taken to be the zero location for gravitational potential energy ($y = 0$). The variable x will represent the amount of spring compression or stretch. We have $v_1 = 0,\ x_1 = 0.50\,\text{m},\ y_1 = 0,\ v_2 = 0,$ and $x_2 = 0.$ The distance the block moves up the plane is given by $d = \dfrac{y}{\sin\theta}$, so $y_2 = d\sin\theta$. Solve for d.

$$E_1 = E_2 \ \rightarrow\ \tfrac{1}{2}mv_1^2 + mgy_1 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_2^2 + mgy_2 + \tfrac{1}{2}kx_2^2 \ \rightarrow$$

$$\tfrac{1}{2}kx_1^2 = mgy_2 = mgd\sin\theta \ \rightarrow\ d = \frac{kx_1^2}{2mg\sin\theta} = \frac{(75\,\text{N/m})(0.50\,\text{m})^2}{2(2.0\,\text{kg})(9.80\,\text{m/s}^2)\sin 41^\circ} = \boxed{0.73\,\text{m}}$$

(*b*) Now the spring will be stretched at the turning point of the motion. The first half-meter of the block's motion returns the block to the equilibrium position of the spring. After that, the block beings to stretch the spring. Accordingly, we have the same conditions as before except that $x_2 = d - 0.5\,\text{m}$.

$$E_1 = E_2 \ \rightarrow\ \tfrac{1}{2}mv_1^2 + mgy_1 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_2^2 + mgy_2 + \tfrac{1}{2}kx_2^2 \ \rightarrow$$

$$\tfrac{1}{2}kx_1^2 = mgd\sin\theta + \tfrac{1}{2}k(d - 0.5\,\text{m})$$

This is a quadratic relation in d. Solving it gives $d = \boxed{0.66\,\text{m}}$.

(*c*) The block now moves $d = 0.50\,\text{m}$, and stops at the equilibrium point of the spring. Accordingly, $x_2 = 0$. Apply the method of Section 8-6.

$$\Delta K + \Delta U + F_{\text{fr}}\ell = \tfrac{1}{2}m\left(v_2^2 - v_1^2\right) + \tfrac{1}{2}k\left(x_2^2 - x_1^2\right) + mg\left(y_2 - y_1\right) + \mu_k mgd\cos\theta \ \rightarrow$$

$$\mu_k = \frac{-\tfrac{1}{2}kx_1^2 + mgd\sin\theta}{-mgd\cos\theta} = \frac{kx_1^2}{2mgd\cos\theta} - \tan\theta$$

$$= \frac{(75\,\text{N/m})(0.50\,\text{m})^2}{2(2.0\,\text{kg})(9.80\,\text{m/s}^2)(0.50\,\text{m})\cos 41^\circ} - \tan 41^\circ = \boxed{0.40}$$

45)

(*a*) Equate the gravitational force to the expression for centripetal force, since the orbit is circular. Let M_E represent the mass of the Earth.

$$\frac{m_s v_s^2}{r_s} = \frac{GM_E m_s}{r_s^2} \ \rightarrow\ m_s v_s^2 = \frac{GM_E m_s}{r_s} \ \rightarrow\ \tfrac{1}{2}m_s v_s^2 = \boxed{K = \frac{GM_E m_s}{2r_s}}$$

(*b*) The potential energy is given by Eq. 8-17, $\boxed{U = -GM_E m_s/r_s}$.

(*c*) $$\frac{K}{U} = \frac{\dfrac{GM_E m_s}{2r_s}}{-\dfrac{GM_E m_s}{r_s}} = \boxed{-\frac{1}{2}}$$

49)

The escape velocity for an object located a distance r from a mass M is given by Eq. 8-19, $v_{esc} = \sqrt{\frac{2MG}{r}}$. The orbit speed for an object located a distance r from a mass M is $v_{orb} = \sqrt{\frac{MG}{r}}$.

(a) $$v_{\substack{\text{esc at}\\ \text{Sun's}\\ \text{surface}}} = \sqrt{\frac{2M_{Sun}G}{r_{Sun}}} = \sqrt{\frac{2\left(2.0\times10^{30}\,\text{kg}\right)\left(6.67\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2\right)}{7.0\times10^{8}\,\text{m}}} = \boxed{6.2\times10^{5}\,\text{m/s}}$$

(b) $$v_{\substack{\text{esc at}\\ \text{Earth}\\ \text{orbit}}} = \sqrt{\frac{2M_{Sun}G}{r_{\text{Earth orbit}}}} = \sqrt{\frac{2\left(2.0\times10^{30}\,\text{kg}\right)\left(6.67\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2\right)}{1.50\times10^{-11}\,\text{m}}} = \boxed{4.2\times10^{4}\,\text{m/s}}$$

$$\frac{v_{\substack{\text{esc at}\\ \text{Earth}\\ \text{orbit}}}}{v_{\substack{\text{Earth}\\ \text{orbit}}}} = \frac{\sqrt{\frac{2M_{Sun}G}{r_{\text{Earth orbit}}}}}{\sqrt{\frac{M_{Sun}G}{r_{\text{Earth orbit}}}}} = \sqrt{2} \rightarrow \boxed{v_{\substack{\text{esc at}\\ \text{Earth}\\ \text{orbit}}} = \sqrt{2}\,v_{\substack{\text{Earth}\\ \text{orbit}}}}$$

Since $v_{\substack{\text{esc at}\\ \text{Earth}\\ \text{orbit}}} \approx 1.4 v_{\substack{\text{Earth}\\ \text{orbit}}}$, the orbiting object will not escape the orbit.

52)

(a) With the condition that $U = 0$ at $r = \infty$, the potential energy is given by $U = -\frac{GM_E m}{r}$. The kinetic energy is found from the fact that for a circular orbit, the gravitational force is a centripetal force.

$$\frac{GM_E m}{r^2} = \frac{mv_{orbit}^2}{r} \rightarrow mv_{orbit}^2 = \frac{GM_E m}{r} \rightarrow K = \tfrac{1}{2}mv_{orbit}^2 = \tfrac{1}{2}\frac{GM_E m}{r}$$

$$E = K + U = \tfrac{1}{2}\frac{GM_E m}{r} - \frac{GM_E m}{r} = \boxed{-\tfrac{1}{2}\frac{GM_E m}{r}}$$

(b) As the value of E decreases, since E is negative, the radius r must get smaller. But as the radius gets smaller, the kinetic energy increases, since $K \propto \frac{1}{r}$. If the total energy decreases by 1 Joule, the potential energy decreases by 2 Joules and the kinetic energy increases by 1 Joule.

57)

The external work required $\left(W_{other}\right)$ is the change in the mechanical energy of the satellite. Note the following, from the work-energy theorem.

$$W_{total} = W_{gravity} + W_{other} \rightarrow \Delta K = -\Delta U + W_{other} \rightarrow W_{other} = \Delta K + \Delta U = \Delta\left(K+U\right) = \Delta E_{mech}$$

From problem 52, we know that the mechanical energy is given by $E = -\tfrac{1}{2}\frac{GMm}{r}$.

$$E = -\tfrac{1}{2}\frac{GMm}{r} \rightarrow \Delta E = \left(-\tfrac{1}{2}\frac{GMm}{r}\right)_{final} - \left(-\tfrac{1}{2}\frac{GMm}{r}\right)_{initial} = \left(\tfrac{1}{2}\frac{GMm}{r}\right)_{initial} - \left(\tfrac{1}{2}\frac{GMm}{r}\right)_{final}$$

$$= \left(\tfrac{1}{2}\frac{GMm}{2r_E}\right)_{initial} - \left(\tfrac{1}{2}\frac{GMm}{3r_E}\right)_{final} = \boxed{\frac{GMm}{12r_E}}$$

62)

The work necessary to lift the piano is the work done by an upward force, equal in magnitude to the weight of the piano. Thus $W = Fd\cos 0° = mgh$. The average power output required to lift the piano is the work done divided by the time to lift the piano.

$$P = \frac{W}{t} = \frac{mgh}{t} \rightarrow t = \frac{mgh}{P} = \frac{(335\text{ kg})(9.80\text{ m/s}^2)(16.0\text{ m})}{1750\text{ W}} = \boxed{30.0\text{ s}}$$

65)

The energy transfer from the engine must replace the lost kinetic energy. From the two speeds, calculate the average rate of loss in kinetic energy while in neutral.

$$v_1 = 95\text{ km/h}\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right) = 26.39\text{ m/s} \qquad v_2 = 65\text{ km/h}\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right) = 18.06\text{ m/s}$$

$$\Delta KE = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}(1080\text{ kg})\left[(18.06\text{ m/s})^2 - (26.39\text{ m/s})^2\right] = -1.999\times10^5\text{ J}$$

$$P = \frac{W}{t} = \frac{1.999\times10^5\text{ J}}{7.0\text{ s}} = 2.856\times10^4\text{ W},\text{ or } (2.856\times10^4\text{ W})\frac{1\text{ hp}}{746\text{ W}} = 38.29\text{ hp}$$

So $\boxed{2.9\times10^4\text{ W}}$ or $\boxed{38\text{ hp}}$ is needed from the engine.

77)

(*a*) To find possible minima and maxima, set the first derivative of the function equal to 0 and solve for the values of r.

$$U(r) = -\frac{a}{r^6} + \frac{b}{r^{12}} = \frac{1}{r^{12}}(b - ar^6) \rightarrow \frac{dU}{dr} = 6\frac{a}{r^7} - 12\frac{b}{r^{13}}$$

$$\frac{dU}{dr} = 0 \rightarrow \frac{a}{r^7} = 2\frac{b}{r^{13}} \rightarrow r_{\text{crit}} = \left(\frac{2b}{a}\right)^{1/6}, \infty$$

The second derivative test is used to determine the actual type of critical points found.

$$\frac{d^2U}{dr^2} = -42\frac{a}{r^8} + 156\frac{b}{r^{14}} = \frac{1}{r^{14}}(156b - 42ar^6)$$

$$\left.\frac{d^2U}{dr^2}\right|_{\left(\frac{2b}{a}\right)^{1/6}} = \frac{1}{\left(\frac{2b}{a}\right)^{14/6}}\left(156b - 42a\frac{2b}{a}\right) = \frac{1}{\left(\frac{2b}{a}\right)^{14/6}}(156b - 84b) > 0 \rightarrow r_{\text{crit}} = \left(\frac{2b}{a}\right)^{1/6}$$

Thus there is a $\boxed{\text{minimum at } r = \left(\frac{2b}{a}\right)^{1/6}}$. We also must check the endpoints of the function.

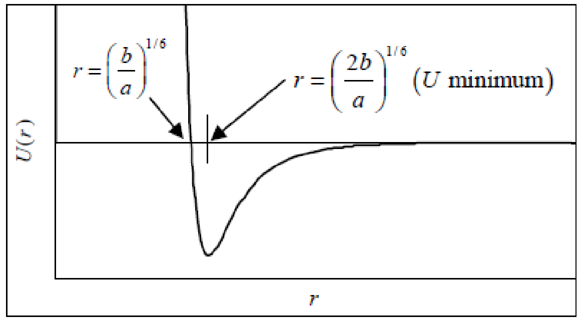
We see from the form $U(r) = \frac{1}{r^{12}}(b - ar^6)$ that as $r \to 0$, $U(r) \to \infty$, and so there is a $\boxed{\text{maximum at } r = 0}$.

(*b*) Solve $U(r) = 0$ for the distance.

$$U(r) = -\frac{a}{r^6} + \frac{b}{r^{12}} = \frac{1}{r^{12}}(b - ar^6) = 0 \rightarrow \frac{1}{r^{12}} = 0 \text{ or } (b - ar^6) = 0 \rightarrow$$

$$\boxed{r = \infty\ ;\ r = \left(\frac{b}{a}\right)^{1/6}}$$

(c) See the adjacent graph.



(d) For $E < 0$, there will be bound oscillatory motion between two turning points. This could represent a chemical bond type of situation. For $E > 0$, the motion will be unbounded, and so the atoms will not stay together.

(e) The force is the opposite of the slope of the potential energy graph.

$$\boxed{F > 0 \text{ for } r < \left(\frac{2b}{a}\right)^{1/6}} \;;\; \boxed{F < 0 \text{ for } \left(\frac{2b}{a}\right)^{1/6} < r < \infty} \;;\; \boxed{F = 0 \text{ for } r = \left(\frac{2b}{a}\right)^{1/6}, r = \infty}$$

(f) $F(r) = -\frac{dU}{dr} = \boxed{\frac{12b}{r^{13}} - \frac{6a}{r^7}}$

92)

(a) $$F(r) = -\frac{dU(r)}{dr} = -\left[(-U_0)\left(-\frac{r_0}{r^2}\right)e^{-r/r_0} + \left(-U_0\frac{r_0}{r}\right)\left(-\frac{1}{r_0}\right)e^{-r/r_0}\right] = \boxed{-U_0\frac{r_0}{r}e^{-r/r_0}\left(\frac{1}{r} + \frac{1}{r_0}\right)}$$

(b) $$F(3r_0)/F(r_0) = \frac{-U_0\frac{r_0}{3r_0}e^{-3r_0/r_0}\left(\frac{1}{3r_0} + \frac{1}{r_0}\right)}{-U_0\frac{r_0}{r_0}e^{-r_0/r_0}\left(\frac{1}{r_0} + \frac{1}{r_0}\right)} = \boxed{\tfrac{2}{9}e^{-2} \approx 0.03}$$

(c) $$F(r) = -\frac{dU(r)}{dr} = -\left[(-C)\left(-\frac{1}{r^2}\right)\right] = -\frac{C}{r^2} \;;\; F(3r_0)/F(r_0) = \frac{-C\frac{1}{(3r_0)^2}}{-C\frac{1}{(r_0)^2}} = \boxed{\tfrac{1}{9} \approx 0.1}$$