PHYS 4A – Mechanics by Prof. Hans Paar – Winter 2016

Chapter 7 Solutions

Prepared by: Thyagarajan V

9)

Since the acceleration of the box is constant, use Eq. 2-12b to find the distance moved. Assume that the box starts from rest.

$$d = x - x_0 = v_0 t + \tfrac{1}{2}at^2 = 0 + \tfrac{1}{2}\left(2.0\,\mathrm{m/s^2}\right)\left(7.0\,\mathrm{s}\right)^2 = 49\,\mathrm{m}$$

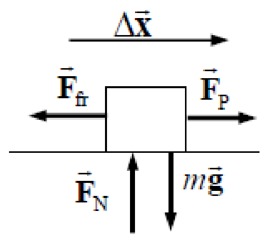
Then the work done in moving the crate is found using Eq. 7-1.

$$W = Fd\cos 0^\circ = mad = \left(6.0\,\mathrm{kg}\right)\left(2.0\,\mathrm{m/s^2}\right)\left(49\,\mathrm{m}\right) = \boxed{590\,\mathrm{J}}$$

14)

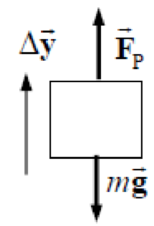
(*a*) See the free-body diagram for the crate as it is being pulled. Since the crate is not accelerating horizontally, $F_P = F_{fr} = 230\,\mathrm{N}$. The work done to move it across the floor is the work done by the pulling force. The angle between the pulling force and the direction of motion is 0°. Use Eq. 7-1.

$$W_P = F_P d\cos 0^\circ = \left(230\ \mathrm{N}\right)\left(4.0\ \mathrm{m}\right)\left(1\right) = \boxed{920\,\mathrm{J}}$$



(*b*) See the free-body diagram for the crate as it is being lifted. Since the crate is not accelerating vertically, the pulling force is the same magnitude as the weight. The angle between the pulling force and the direction of motion is 0°.

$$W_P = F_P d\cos 0^\circ = mgd = \left(2200\,\mathrm{N}\right)\left(4.0\,\mathrm{m}\right) = \boxed{8800\,\mathrm{J}}$$



16)

Use Eq. 7.4 to calculate the dot product.

$$\vec{\mathbf{A}}\bullet\vec{\mathbf{B}} = A_x B_x + A_y B_y + A_z B_z = \left(2.0x^2\right)\left(11.0\right) + \left(-4.0x\right)\left(2.5x\right) + \left(5.0\right)\left(0\right) = 22x^2 - 10x^2$$

$$= \boxed{12x^2}$$

17)

Use Eq. 7.4 to calculate the dot product. Note that $\hat{\mathbf{i}} = 1\hat{\mathbf{i}} + 0\hat{\mathbf{j}} + 0\hat{\mathbf{k}}$, $\hat{\mathbf{j}} = 0\hat{\mathbf{i}} + 1\hat{\mathbf{j}} + 0\hat{\mathbf{k}}$, and $\hat{\mathbf{k}} = 0\hat{\mathbf{i}} + 0\hat{\mathbf{j}} + 1\hat{\mathbf{k}}$.

$$\hat{\mathbf{i}}\bullet\vec{\mathbf{V}} = (1)V_x + (0)V_y + (0)V_z = V_x \qquad \hat{\mathbf{j}}\bullet\vec{\mathbf{V}} = (0)V_x + (1)V_y + (0)V_z = V_y$$

$$\hat{\mathbf{k}}\bullet\vec{\mathbf{V}} = (0)V_x + (0)V_y + (1)V_z = V_z$$

21)

If $\vec{\mathbf{A}}$ is perpendicular to $\vec{\mathbf{B}}$, then $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = 0$. Use this to find $\vec{\mathbf{B}}$.

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = A_x B_x + A_y B_y = (3.0) B_x + (1.5) B_y = 0 \quad \rightarrow \quad B_y = -2.0 B_x$$

Any vector $\vec{\mathbf{B}}$ that satisfies $B_y = -2.0 B_x$ will be perpendicular to $\vec{\mathbf{A}}$. For example, $\vec{\mathbf{B}} = \boxed{1.5\hat{\mathbf{i}} - 3.0\hat{\mathbf{j}}}$.

27)

Note that by Eq. 7-2, the dot product of a vector $\vec{\mathbf{A}}$ with a unit vector $\vec{\mathbf{B}}$ would give the magnitude of $\vec{\mathbf{A}}$ times the cosine of the angle between the unit vector and $\vec{\mathbf{A}}$. Thus if the unit vector lies along one of the coordinate axes, we can find the angle between the vector and the coordinate axis. We also use Eq. 7-4 to give a second evaluation of the dot product.

$$\vec{\mathbf{V}} \cdot \hat{\mathbf{i}} = V \cos\theta_x = V_x \quad \rightarrow$$

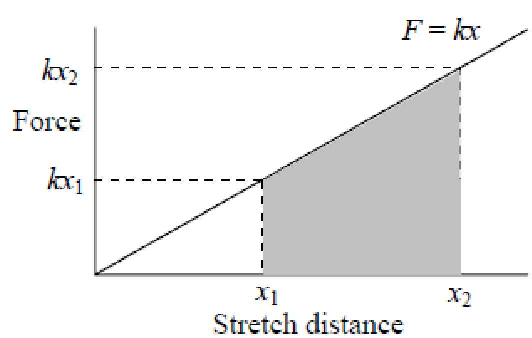
$$\theta_x = \cos^{-1}\frac{V_x}{V} = \cos^{-1}\frac{V_x}{\sqrt{V_x^2 + V_y^2 + V_z^2}} = \cos^{-1}\frac{20.0}{\sqrt{(20.0)^2 + (22.0)^2 + (-14.0)^2}} = \boxed{52.5^\circ}$$

$$\theta_y = \cos^{-1}\frac{V_y}{V} = \cos^{-1}\frac{22.0}{\sqrt{(20.0)^2 + (22.0)^2 + (-14.0)^2}} = \boxed{48.0^\circ}$$

$$\theta_z = \cos^{-1}\frac{V_z}{V} = \cos^{-1}\frac{-14.0}{\sqrt{(20.0)^2 + (22.0)^2 + (-14.0)^2}} = \boxed{115^\circ}$$

35)

The force exerted to stretch a spring is given by $F_{\text{stretch}} = kx$ (the opposite of the force exerted by the spring, which is given by $F = -kx$. A graph of F_{stretch} vs. x will be a straight line of slope k through the origin. The stretch from x_1 to x_2, as shown on the graph, outlines a trapezoidal area. This area represents the work.



$$W = \tfrac{1}{2}(kx_1 + kx_2)(x_2 - x_1) = \tfrac{1}{2}k(x_1 + x_2)(x_2 - x_1)$$

$$= \tfrac{1}{2}(65\,\text{N/m})(0.095\,\text{m})(0.035\,\text{m}) = \boxed{0.11\,\text{J}}$$

40)

The work done will be the area under the F_x vs. x graph.

(*a*) From $x = 0.0$ to $x = 10.0\,\text{m}$, the shape under the graph is trapezoidal. The area is

$$W_a = (400\,\text{N})\tfrac{1}{2}(10\,\text{m} + 4\,\text{m}) = \boxed{2800\,\text{J}}.$$

(*b*) From $x = 10.0\,\text{m}$ to $x = 15.0\,\text{m}$, the force is in the opposite direction from the direction of motion, and so the work will be negative. Again, since the shape is trapezoidal, we find

$$W_a = (-200\,\text{N})\tfrac{1}{2}(5\,\text{m} + 2\,\text{m}) = -700\,\text{J}.$$

Thus the total work from $x = 0.0$ to $x = 15.0\,\text{m}$ is $2800\text{ J} - 700\text{ J} = \boxed{2100\,\text{J}}$.

45)

Integrate the force over the distance the force acts to find the work.

$$W = \int_0^{1.0\,\text{m}} F_1 dx = \int_0^{1.0\,\text{m}} \frac{A}{\sqrt{x}} dx = 2A\sqrt{x}\Big|_0^{1.0\,\text{m}} = 2\left(2.0\,\text{N}\cdot\text{m}^{1/2}\right)(1.0\,\text{m})^{1/2} = \boxed{4.0\,\text{J}}$$

Note that the work done is finite.

47)

Since the force is of constant magnitude and always directed at $30°$ to the displacement, we have a simple expression for the work done as the object moves.

$$W = \int_{\text{start}}^{\text{finish}} \vec{\mathbf{F}}\cdot d\vec{\ell} = \int_{\text{start}}^{\text{finish}} F\cos 30° d\ell = F\cos 30° \int_{\text{start}}^{\text{finish}} d\ell = F\cos 30° \pi R = \boxed{\frac{\sqrt{3}\pi FR}{2}}$$

63)

(*a*) The angle between the pushing force and the displacement is $32°$.

$$W_{\text{P}} = F_{\text{P}} d\cos\theta = (150\,\text{N})(5.0\,\text{m})\cos 32° = 636.0\,\text{J} \approx \boxed{640\,\text{J}}$$

(*b*) The angle between the force of gravity and the displacement is $122°$.

$$W_{\text{G}} = F_{\text{G}} d\cos\theta = mgd\cos\theta = (18\,\text{kg})\left(9.80\,\text{m/s}^2\right)(5.0\,\text{m})\cos 122° = -467.4\,\text{J} \approx \boxed{-470\,\text{J}}$$

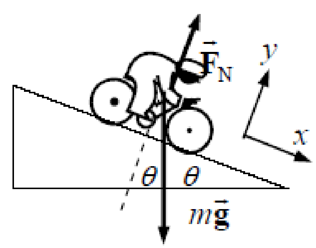
(*c*) Because the normal force is perpendicular to the displacement, the work done by the normal force is $\boxed{0}$.

(*d*) The net work done is the change in kinetic energy.

$$W = W_{\text{P}} + W_{\text{g}} + W_{\text{N}} = \Delta K = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 \quad \rightarrow$$

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(636.0\,\text{J} - 467.4\,\text{J})}{(18\,\text{kg})}} = \boxed{4.3\,\text{m/s}}$$

89)



(*a*) The work done by gravity is given by Eq. 7-1.

$$W_{\text{G}} = mgd\cos(90-\theta) = (85\,\text{kg})\left(9.80\,\text{m/s}^2\right)(250\,\text{m})\cos 86.0°$$

$$= 1.453\times 10^4\,\text{J} \approx \boxed{1.5\times 10^4\,\text{J}}$$

(*b*) The work is the change in kinetic energy. The initial kinetic energy is 0.

$$W_{\text{G}} = \Delta K = K_{\text{f}} - K_{\text{i}} = \tfrac{1}{2}mv_{\text{f}}^2 \quad \rightarrow \quad v_f = \sqrt{\frac{2W_{\text{G}}}{m}} = \sqrt{\frac{2\left(1.453\times 10^4\,\text{J}\right)}{85\,\text{kg}}} = \boxed{18\,\text{m/s}}$$