PHYS 4A – Mechanics by Prof. Hans Paar – Winter 2016

Homework 3 Solutions

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Chapter 6 problems

11)

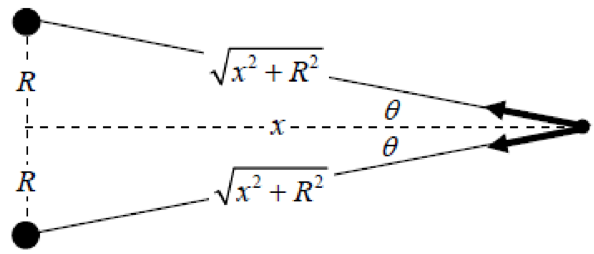
The force on m due to $2m$ points in the $\hat{\mathbf{i}}$ direction. The force on m due to $4m$ points in the $\hat{\mathbf{j}}$ direction. The force on m due to $3m$ points in the direction given by $\theta = \tan^{-1}\frac{y_0}{x_0}$. Add the force vectors together to find the net force.

$$\vec{\mathbf{F}} = G\frac{(2m)m}{x_0^2}\hat{\mathbf{i}} + G\frac{(4m)m}{y_0^2}\hat{\mathbf{j}} + G\frac{(3m)m}{x_0^2+y_0^2}\cos\theta\,\hat{\mathbf{i}} + G\frac{(3m)m}{x_0^2+y_0^2}\sin\theta\,\hat{\mathbf{j}}$$

$$= G\frac{2m^2}{x_0^2}\hat{\mathbf{i}} + G\frac{4m^2}{y_0^2}\hat{\mathbf{j}} + G\frac{3m^2}{x_0^2+y_0^2}\frac{x_0}{\sqrt{x_0^2+y_0^2}}\hat{\mathbf{i}} + G\frac{(3m)m}{x_0^2+y_0^2}\frac{y_0}{\sqrt{x_0^2+y_0^2}}\hat{\mathbf{j}}$$

$$= Gm^2\left[\left(\frac{2}{x_0^2} + \frac{3x_0}{\left(x_0^2+y_0^2\right)^{3/2}}\right)\hat{\mathbf{i}} + \left(\frac{4}{y_0^2} + \frac{3y_0}{\left(x_0^2+y_0^2\right)^{3/2}}\right)\hat{\mathbf{j}}\right]$$

17)



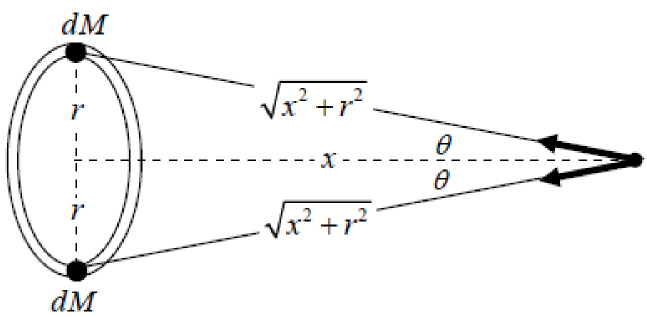
Each mass M will exert a gravitational force on mass m. The vertical components of the two forces will sum to be 0, and so the net force on m is directed horizontally. That net force will be twice the horizontal component of either force.

$$F_{Mm} = \frac{GMm}{\left(x^2+R^2\right)} \quad \rightarrow$$

$$F_{Mm\,x} = \frac{GMm}{\left(x^2+R^2\right)}\cos\theta = \frac{GMm}{\left(x^2+R^2\right)}\frac{x}{\sqrt{x^2+R^2}} = \frac{GMmx}{\left(x^2+R^2\right)^{3/2}}$$

$$F_{\text{net}\,x} = 2F_{Mm\,x} = \boxed{\frac{2GMmx}{\left(x^2+R^2\right)^{3/2}}}$$

18)



From the symmetry of the problem, we can examine diametrically opposite infinitesimal masses and see that only the horizontal components of the force will be left. Any off-axis components of force will add to zero. The infinitesimal horizontal force on m due to an infinitesimal mass dM is $dF_{dMm} = \frac{Gm}{\left(x^2+r^2\right)}dM$.

The horizontal component of that force is given by the following.

$$\left(dF_{dMm}\right)_x = \frac{Gm}{\left(x^2+r^2\right)}\cos\theta dM = \frac{Gm}{\left(x^2+r^2\right)}\frac{x}{\sqrt{\left(x^2+r^2\right)}}dM = \frac{Gmx}{\left(x^2+r^2\right)^{3/2}}dM$$

The total force is then found by integration.

$$dF_x = \frac{Gmx\,dM}{\left(x^2+r^2\right)^{3/2}} \rightarrow \int dF_x = \int\frac{Gmx\,dM}{\left(x^2+r^2\right)^{3/2}} \rightarrow F_x = \boxed{\frac{GMmx}{\left(x^2+r^2\right)^{3/2}}}$$

From the diagram we see that it points inward towards the center of the ring.

19)

The expression for g at the surface of the Earth is $g = G\frac{m_E}{r_E^2}$. Let $g+\Delta g$ be the value at a distance of $r_E+\Delta r$ from the center of Earth, which is Δr above the surface.

(a) $$g = G\frac{m_E}{r_E^2} \rightarrow g+\Delta g = G\frac{m_E}{\left(r_E+\Delta r\right)^2} = G\frac{m_E}{r_E^2\left(1+\frac{\Delta r}{r_E}\right)^2} = G\frac{m_E}{r_E^2}\left(1+\frac{\Delta r}{r_E}\right)^{-2} \approx g\left(1-2\frac{\Delta r}{r_E}\right) \rightarrow$$

$$\boxed{\Delta g \approx -2g\frac{\Delta r}{r_E}}$$

(b) The minus sign indicated that the change in g is in the opposite direction as the change in r. So, if r increases, g decreases, and vice-versa.

(c) Using this result:

$$\Delta g \approx -2g\frac{\Delta r}{r_E} = -2\left(9.80\,\text{m/s}^2\right)\frac{1.25\times10^5\,\text{m}}{6.38\times10^6\,\text{m}} = -0.384\,\text{m/s}^2 \rightarrow g = \boxed{9.42\,\text{m/s}^2}$$

Direct calculation:

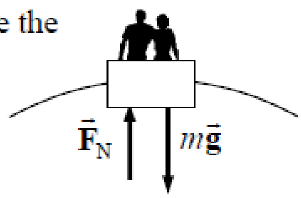
$$g = G\frac{m_E}{r^2} = \left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\frac{\left(5.98\times10^{24}\,\text{kg}\right)}{\left(6.38\times10^6\,\text{m}+1.25\times10^5\,\text{m}\right)^2} = 9.43\,\text{m/s}^2$$

The difference is only about 0.1%.

32)

The apparent weight is the normal force on the passenger. For a person at rest, the normal force is equal to the actual weight. If there is acceleration in the vertical direction, either up or down, then the normal force (and hence the apparent weight) will be different than the actual weight. The speed of the Ferris wheel is $v = 2\pi r/T = 2\pi(11.0\,\text{m})/12.5\,\text{s} = 5.529\,\text{m/s}$.

(a) See the free-body diagram for the highest point of the motion. We assume the passengers are right-side up, so that the normal force of the Ferris wheel seat is upward. The net force must point to the center of the circle, so write Newton's second law with downward as the positive direction. The acceleration is centripetal since the passengers are moving in a circle.



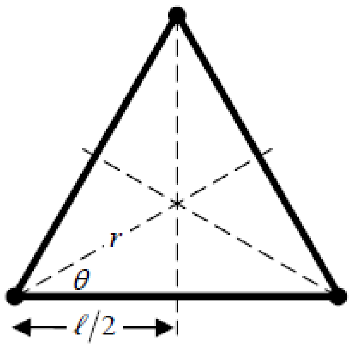
$$\sum F = F_R = mg - F_N = ma = mv^2/r \rightarrow F_N = mg - mv^2/r$$

The ratio of apparent weight to real weight is given by the following.

$$\frac{mg - mv^2/r}{mg} = \frac{g - v^2/r}{g} = 1 - \frac{v^2}{rg} = 1 - \frac{(5.529\,\text{m/s})^2}{(11.0\,\text{m})\left(9.80\,\text{m/s}^2\right)} = \boxed{0.716}$$

35)

Consider the lower left mass in the diagram. The center of the orbits is the intersection of the three dashed lines in the diagram. The net force on the lower left mass is the vector sum of the forces from the other two masses, and points to the center of the orbits. To find that net force, project each force to find the component that lies along the line towards the center. The angle is $\theta = 30^\circ$.



$$F = G\frac{M^2}{\ell^2} \rightarrow F_{\substack{\text{component}\\\text{towards}\\\text{center}}} = F\cos\theta = G\frac{M^2}{\ell^2}\frac{\sqrt{3}}{2} \rightarrow$$

$$F_{\text{net}} = 2G\frac{M^2}{\ell^2}\frac{\sqrt{3}}{2} = \sqrt{3}G\frac{M^2}{\ell^2}$$

The net force is causing centripetal motion, and so is of the form Mv^2/r. Note that $r\cos\theta = \ell/2$.

$$F_{\text{net}} = 2G\frac{M^2}{\ell^2}\frac{\sqrt{3}}{2} = \sqrt{3}G\frac{M^2}{\ell^2} = \frac{Mv^2}{r} = \frac{Mv^2}{\ell/(2\cos\theta)} = \frac{Mv^2}{\ell/\sqrt{3}} \rightarrow \sqrt{3}G\frac{M^2}{\ell^2} = \frac{Mv^2}{\ell/\sqrt{3}} \rightarrow$$

$$v = \boxed{\sqrt{\frac{GM}{\ell}}}$$

40)

We have already seen the speed for an object orbiting a distance r around a mass M is given by $v = \sqrt{GM/r}$.

$$\frac{v_A}{v_B} = \frac{\sqrt{\dfrac{GM_{\text{star}}}{r_A}}}{\sqrt{\dfrac{GM_{\text{star}}}{r_B}}} = \sqrt{\frac{r_B}{r_A}} = \sqrt{\frac{1}{9}} = \boxed{\frac{1}{3}}$$

53)

(a) The acceleration due to gravity at any location at or above the surface of a star is given by $g_{\text{star}} = GM_{\text{star}}/r^2$, where r is the distance from the center of the star to the location in question.

$$g_{\text{star}} = G\frac{M_{\text{sun}}}{R_{\text{Moon}}^2} = \left(6.67\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2\right)\frac{\left(1.99\times10^{30}\,\text{kg}\right)}{\left(1.74\times10^{6}\,\text{m}\right)^2} = \boxed{4.38\times10^{7}\,\text{m/s}^2}$$

(b) $W = mg_{\text{star}} = (65\,\text{kg})\left(4.38\times10^7\,\text{m/s}^2\right) = \boxed{2.8\times10^9\,\text{N}}$

(c) Use Eq. 2-12c, with an initial velocity of 0.

$$v^2 = v_0^2 + 2a(x - x_0) \rightarrow$$

$$v = \sqrt{2a(x-x_0)} = \sqrt{2\left(4.38\times10^7\,\text{m/s}^2\right)(1.0\,\text{m})} = \boxed{9.4\times10^3\,\text{m/s}}$$

77)

Equate the force of gravity on a mass m at the surface of the Earth as expressed by the acceleration due to gravity to that as expressed by Newton's law of universal gravitation.

$$mg = \frac{GM_{\text{Earth}}m}{R_{\text{Earth}}^2} \quad \rightarrow \quad G = \frac{gR_{\text{Earth}}^2}{M_{\text{Earth}}} = \frac{gR_{\text{Earth}}^2}{\rho_{\text{Earth}}\frac{4}{3}\pi R_{\text{Earth}}^3} = \frac{3g}{4\pi\rho R_{\text{Earth}}} = \frac{3g}{4\pi\rho\frac{C_{\text{Earth}}}{2\pi}} = \frac{3g}{2\rho C_{\text{Earth}}}$$

$$= \frac{3\left(10\,\text{m/s}^2\right)}{2\left(3000\,\text{kg/m}^3\right)\left(4\times10^7\,\text{m}\right)} = 1.25\times10^{-10}\,\frac{\text{m}^3}{\text{kg}\cdot\text{s}^2} \approx \boxed{1\times10^{-10}\,\frac{\text{m}^3}{\text{kg}\cdot\text{s}^2}}$$

This is roughly twice the size of the accepted value of G.