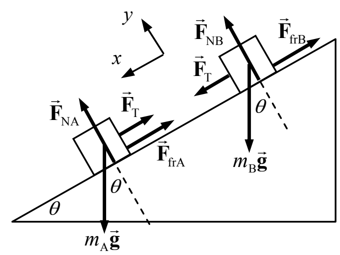
UCSD Phys 4A – Intro Mechanics – Winter 2016
Ch 5 Solutions

20. Since the upper block has a higher coefficient of friction, that block will “drag behind” the lower block. Thus there will be tension in the cord, and the blocks will have the same acceleration. From the free-body diagrams for each block, we write Newton’s second law for both the x and y directions for each block, and then combine those equations to find the acceleration and tension.



(*a*) Block A:

$$\sum F_{yA} = F_{NA} - m_A g\cos\theta = 0 \;\rightarrow\; F_{NA} = m_A g\cos\theta$$

$$\sum F_{xA} = m_A g\sin\theta - F_{frA} - F_T = m_A a$$

$$m_A a = m_A g\sin\theta - \mu_A F_{NA} - F_T = m_A g\sin\theta - \mu_A m_A g\cos\theta - F_T$$

Block B:

$$\sum F_{yB} = F_{NB} - m_B g\cos\theta = 0 \;\rightarrow\; F_{NB} = m_B g\cos\theta$$

$$\sum F_{xB} = m_A g\sin\theta - F_{frA} + F_T = m_B a$$

$$m_B a = m_B g\sin\theta - \mu_B F_{NB} + F_T = m_B g\sin\theta - \mu_B m_B g\cos\theta + F_T$$

Add the final equations together from both analyses and solve for the acceleration.

$$m_A a = m_A g\sin\theta - \mu_A m_A g\cos\theta - F_T \;\;;\;\; m_B a = m_B g\sin\theta - \mu_B m_B g\cos\theta + F_T$$

$$m_A a + m_B a = m_A g\sin\theta - \mu_A m_A g\cos\theta - F_T + m_B g\sin\theta - \mu_B m_B g\cos\theta + F_T \;\rightarrow$$

$$a = g\left[\frac{m_A(\sin\theta - \mu_A\cos\theta) + m_B(\sin\theta - \mu_B\cos\theta)}{(m_A + m_B)}\right]$$

$$= (9.80\,\mathrm{m/s^2})\left[\frac{(5.0\,\mathrm{kg})(\sin 32^\circ - 0.20\cos 32^\circ) + (5.0\,\mathrm{kg})(\sin 32^\circ - 0.30\cos 32^\circ)}{(10.0\,\mathrm{kg})}\right]$$

$$= 3.1155\,\mathrm{m/s^2} \approx \boxed{3.1\,\mathrm{m/s^2}}$$

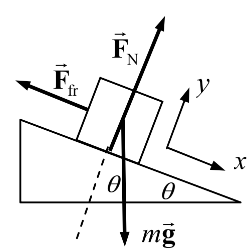
(*b*) Solve one of the equations for the tension force.

$$m_A a = m_A g\sin\theta - \mu_A m_A g\cos\theta - F_T \;\rightarrow$$

$$F_T = m_A(g\sin\theta - \mu_A g\cos\theta - a)$$

$$= (5.0\,\mathrm{kg})\left[(9.80\,\mathrm{m/s^2})(\sin 32^\circ - 0.20\cos 32^\circ) - 3.1155\,\mathrm{m/s^2}\right] = \boxed{2.1\,\mathrm{N}}$$

18. (*a*) Consider the free-body diagram for the crate on the surface. There is no motion in the y direction and thus no acceleration in the y direction. Write Newton’s second law for both directions.



$$\sum F_y = F_N - mg\cos\theta = 0 \;\rightarrow\; F_N = mg\cos\theta$$

$$\sum F_x = mg\sin\theta - F_{fr} = ma$$

$$ma = mg\sin\theta - \mu_k F_N = mg\sin\theta - \mu_k mg\cos\theta$$

$$a = g(\sin\theta - \mu_k\cos\theta)$$

$$= (9.80\,\mathrm{m/s^2})(\sin 25.0^\circ - 0.19\cos 25.0^\circ) = 2.454\,\mathrm{m/s^2} \approx \boxed{2.5\,\mathrm{m/s^2}}$$

(*b*) Now use Eq. 2-12c, with an initial velocity of 0, to find the final velocity.

$$v^2 - v_0^2 = 2a(x - x_0) \;\rightarrow\; v = \sqrt{2a(x - x_0)} = \sqrt{2(2.454\,\mathrm{m/s^2})(8.15\,\mathrm{m})} = \boxed{6.3\,\mathrm{m/s}}$$

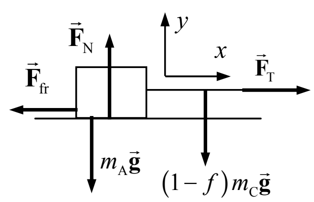
23. (*a*) For m_B to not move, the tension must be equal to $m_B g$, and so $m_B g = F_T$. For m_A to not move, the tension must be equal to the force of static friction, and so $F_S = F_T$. Note that the normal force on m_A is equal to its weight. Use these relationships to solve for m_A.

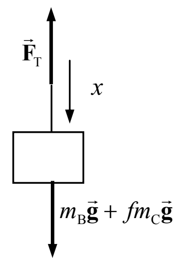
$$m_B g = F_T = F_s \le \mu_s m_A g \quad \rightarrow \quad m_A \ge \frac{m_B}{\mu_s} = \frac{2.0\,\text{kg}}{0.40} = 5.0\text{ kg} \quad \rightarrow \quad m_A \ge \boxed{5.0\text{ kg}}$$

(*b*) For m_B to move with constant velocity, the tension must be equal to $m_B g$. For m_A to move with constant velocity, the tension must be equal to the force of kinetic friction. Note that the normal force on m_A is equal to its weight. Use these relationships to solve for m_A.

$$m_B g = F_k = \mu_k m_A g \quad \rightarrow \quad m_A = \frac{m_B}{\mu_k} = \frac{2.0\,\text{kg}}{0.30} = \boxed{6.7\,\text{kg}}$$

24. We define *f* to be the fraction of the cord that is handing down, between m_B and the pulley. Thus the mass of that piece of cord is fm_C. We assume that the system is moving to the right as well. We take the tension in the cord to be F_T at the pulley. We treat the hanging mass and hanging fraction of the cord as one mass, and the sliding mass and horizontal part of the cord as another mass. See the free-body diagrams. We write Newton's second law for each object.





$$\sum F_{yA} = F_N - \left(m_A + (1-f)m_C\right)g = 0$$

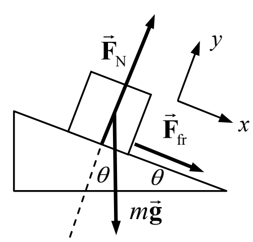
$$\sum F_{xA} = F_T - F_{fr} = F_T - \mu_k F_N = \left(m_A + (1-f)m_C\right)a$$

$$\sum F_{xB} = \left(m_B + fm_C\right)g - F_T = \left(m_B + fm_C\right)a$$

Combine the relationships to solve for the acceleration. In particular, add the two equations for the *x*-direction, and then substitute the normal force.

$$a = \boxed{\left[\frac{m_B + fm_C - \mu_k\left(m_A + (1-f)m_C\right)}{m_A + m_B + m_C}\right]g}$$

$\boxed{25.}$ (*a*) Consider the free-body diagram for the block on the surface. There is no motion in the *y* direction and thus no acceleration in the *y* direction. Write Newton's second law for both directions, and find the acceleration.



$$\sum F_y = F_N - mg\cos\theta = 0 \quad \rightarrow \quad F_N = mg\cos\theta$$

$$\sum F_x = mg\sin\theta + F_{fr} = ma$$

$$ma = mg\sin\theta + \mu_k F_N = mg\sin\theta + \mu_k mg\cos\theta$$

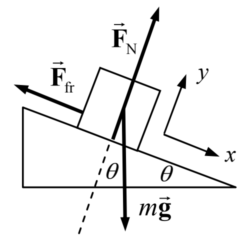
$$a = g\left(\sin\theta + \mu_k\cos\theta\right)$$

Now use Eq. 2-12c, with an initial velocity of v_0, a final velocity of 0, and a displacement of $-d$ to find the coefficient of kinetic friction.

$$v^2 - v_0^2 = 2a(x - x_0) \quad \to \quad 0 - v_0^2 = 2g(\sin\theta + \mu_k \cos\theta)(-d) \quad \to$$

$$\mu_k = \boxed{\frac{v_0^2}{2gd\cos\theta} - \tan\theta}$$

(*b*) Now consider the free-body diagram for the block at the top of its motion. We use a similar force analysis, but now the magnitude of the friction force is given by $F_{\mathrm{fr}} \le \mu_s F_{\mathrm{N}}$, and the acceleration is 0.

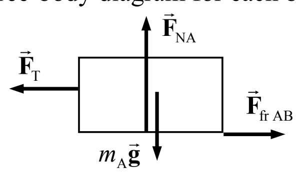


$$\sum F_y = F_{\mathrm{N}} - mg\cos\theta = 0 \quad \to \quad F_{\mathrm{N}} = mg\cos\theta$$

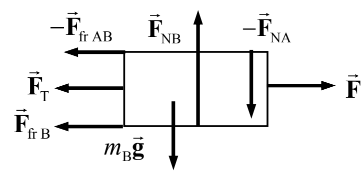
$$\sum F_x = mg\sin\theta - F_{\mathrm{fr}} = ma = 0 \quad \to \quad F_{\mathrm{fr}} = mg\sin\theta$$

$$F_{\mathrm{fr}} \le \mu_s F_{\mathrm{N}} \quad \to \quad mg\sin\theta \le \mu_s mg\cos\theta \quad \to \quad \boxed{\mu_s \ge \tan\theta}$$

$\boxed{31.}$ Draw a free-body diagram for each block.



Block A (top)



Block B (bottom)

$\vec{\mathbf{F}}_{\text{fr AB}}$ is the force of friction between the two blocks, $\vec{\mathbf{F}}_{\text{NA}}$ is the normal force of contact between the two blocks, $\vec{\mathbf{F}}_{\text{fr B}}$ is the force of friction between the bottom block and the floor, and $\vec{\mathbf{F}}_{\text{NB}}$ is the normal force of contact between the bottom block and the floor.

Neither block is accelerating vertically, and so the net vertical force on each block is zero.

top: $F_{\text{NA}} - m_{\text{A}}g = 0 \quad \rightarrow \quad F_{\text{NA}} = m_{\text{A}}g$

bottom: $F_{\text{NB}} - F_{\text{NA}} - m_{\text{B}}g = 0 \quad \rightarrow \quad F_{\text{NB}} = F_{\text{NA}} + m_{\text{B}}g = (m_{\text{A}} + m_{\text{B}})g$

Take the positive horizontal direction to be the direction of motion of each block. Thus for the bottom block, positive is to the right, and for the top block, positive is to the left. Then, since the blocks are constrained to move together by the connecting string, both blocks will have the same acceleration. Write Newton's second law for the horizontal direction for each block.

top: $F_{\text{T}} - F_{\text{fr AB}} = m_{\text{A}}a$ bottom: $F - F_{\text{T}} - F_{\text{fr AB}} - F_{\text{fr B}} = m_{\text{B}}a$

(*a*) If the two blocks are just to move, then the force of static friction will be at its maximum, and so the frictions forces are as follows.

$$F_{\text{fr AB}} = \mu_{\text{s}}F_{\text{NA}} = \mu_{\text{s}}m_{\text{A}}g \quad ; \quad F_{\text{fr B}} = \mu_{\text{s}}F_{\text{NB}} = \mu_{\text{s}}(m_{\text{A}} + m_{\text{B}})g$$

Substitute into Newton's second law for the horizontal direction with $a = 0$ and solve for F.

top: $F_{\text{T}} - \mu_{\text{s}}m_{\text{A}}g = 0 \quad \rightarrow \quad F_{\text{T}} = \mu_{\text{s}}m_{\text{A}}g$

bottom: $F - F_{\text{T}} - \mu_{\text{s}}m_{\text{A}}g - \mu_{\text{s}}(m_{\text{A}} + m_{\text{B}})g = 0 \quad \rightarrow$

$$F = F_{\text{T}} + \mu_{\text{s}}m_{\text{A}}g + \mu_{\text{s}}(m_{\text{A}} + m_{\text{B}})g = \mu_{\text{s}}m_{\text{A}}g + \mu_{\text{s}}m_{\text{A}}g + \mu_{\text{s}}(m_{\text{A}} + m_{\text{B}})g$$

$$= \mu_{\text{s}}(3m_{\text{A}} + m_{\text{B}})g = (0.60)(14\,\text{kg})(9.80\,\text{m/s}^2) = 82.32\,\text{N} \approx \boxed{82\,\text{N}}$$

(*b*) Multiply the force by 1.1 so that $F = 1.1(82.32\,\text{N}) = 90.55\,\text{N}$. Again use Newton's second law for the horizontal direction, but with $a \neq 0$ and using the coefficient of kinetic friction.

top: $F_{\text{T}} - \mu_{\text{k}}m_{\text{A}}g = m_{\text{A}}a$

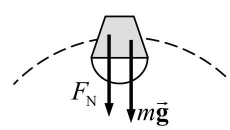
bottom: $F - F_{\text{T}} - \mu_{\text{k}}m_{\text{A}}g - \mu_{\text{k}}(m_{\text{A}} + m_{\text{B}})g = m_{\text{B}}a$

sum: $F - \mu_{\text{k}}m_{\text{A}}g - \mu_{\text{k}}m_{\text{A}}g - \mu_{\text{k}}(m_{\text{A}} + m_{\text{B}})g = (m_{\text{A}} + m_{\text{B}})a \quad \rightarrow$

$$a = \frac{F - \mu_{\text{k}}m_{\text{A}}g - \mu_{\text{k}}m_{\text{A}}g - \mu_{\text{k}}(m_{\text{A}} + m_{\text{B}})g}{(m_{\text{A}} + m_{\text{B}})} = \frac{F - \mu_{\text{k}}(3m_{\text{A}} + m_{\text{B}})g}{(m_{\text{A}} + m_{\text{B}})}$$

$$= \frac{90.55\,\text{N} - (0.40)(14.0\,\text{kg})(9.80\,\text{m/s}^2)}{(8.0\,\text{kg})} = 4.459\,\text{m/s}^2 \approx \boxed{4.5\,\text{m/s}^2}$$

$\boxed{37.}$ We assume the water is rotating in a vertical circle of radius *r*. When the bucket is at the top of its motion, there would be two forces on the water (considering the water as a single mass). The weight of the water would be directed down, and the normal force of the bottom of the bucket pushing on the water would also be down. See the free-body diagram. If the water is moving in a circle, then the net downward force would be a centripetal force.



$$\sum F = F_{\text{N}} + mg = ma = mv^2/r \quad \rightarrow \quad F_{\text{N}} = m(v^2/r - g)$$

The limiting condition of the water falling out of the bucket means that the water loses contact with the bucket, and so the normal force becomes 0.

$$F_{\text{N}} = m(v^2/r - g) \quad \rightarrow \quad m(v_{\text{critical}}^2/r - g) = 0 \quad \rightarrow \quad v_{\text{critical}} = \sqrt{rg}$$

From this, we see that $\boxed{\text{yes}}$, it is possible to whirl the bucket of water fast enough. The minimum speed is $\boxed{\sqrt{rg}}$.

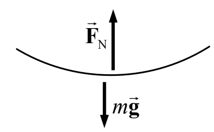
$\boxed{43.}$ The orbit radius will be the sum of the Earth's radius plus the 400 km orbit height. The orbital period is about 90 minutes. Find the centripetal acceleration from these data.

$$r = 6380\,\text{km} + 400\,\text{km} = 6780\,\text{km} = 6.78\times10^{6}\,\text{m} \qquad T = 90\ \text{min}\left(\frac{60\ \text{sec}}{1\ \text{min}}\right) = 5400\ \text{sec}$$

$$a_R = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2\left(6.78\times10^{6}\,\text{m}\right)}{\left(5400\ \text{sec}\right)^2} = \left(9.18\,\text{m/s}^2\right)\left(\frac{1\,g}{9.80\,\text{m/s}^2}\right) = 0.937 \approx \boxed{0.9\,g\text{'s}}$$

Notice how close this is to g, because the shuttle is not very far above the surface of the Earth, relative to the radius of the Earth.

47. (*a*) See the free-body diagram for the pilot in the jet at the bottom of the loop. We have $a_R = v^2/r = 6g$.



$$v^2/r = 6.0g \;\rightarrow\; r = \frac{v^2}{6.0g} = \frac{\left[\left(1200\,\text{km/h}\right)\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2}{6.0\left(9.80\,\text{m/s}^2\right)} = \boxed{1900\,\text{m}}$$

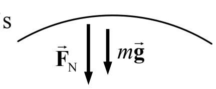
(*b*) The net force must be centripetal, to make the pilot go in a circle. Write Newton's second law for the vertical direction, with up as positive. The normal force is the apparent weight.

$$\sum F_R = F_N - mg = mv^2/r$$

The centripetal acceleration is to be $v^2/r = 6.0g$.

$$F_N = mg + mv^2/r = 7mg = 7\left(78\ \text{kg}\right)\left(9.80\,\text{m/s}^2\right) = 5350\,\text{N} = \boxed{5400\,\text{N}}$$

(*c*) See the free-body diagram for the pilot at the top of the loop. Notice that the normal force is down, because the pilot is upside down. Write Newton's second law in the vertical direction, with down as positive.



$$\sum F_R = F_N + mg = mv^2/r = 6mg \;\rightarrow\; F_N = 5mg = \boxed{3800\,\text{N}}$$

60. (*a*) The object has a uniformly increasing speed, which means the tangential acceleration is constant, and so constant acceleration relationships can be used for the tangential motion. The object is moving in a circle of radius 2.0 meters.

$$\Delta x_{\tan} = \frac{v_{\tan} + v_{0_{\tan}}}{2}t \;\rightarrow\; v_{\tan} = \frac{2\Delta x_{\tan}}{t} - v_{0_{\tan}} = \frac{2\left[\frac{1}{4}\left(2\pi r\right)\right]}{t} = \frac{\pi\left(2.0\,\text{m}\right)}{2.0\,\text{s}} = \boxed{\pi\,\text{m/s}}$$

(*b*) The initial location of the object is at $2.0\,\text{m}\hat{\mathbf{j}}$, and the final location is $2.0\,\text{m}\hat{\mathbf{i}}$.

$$\vec{\mathbf{v}}_{\text{avg}} = \frac{\vec{\mathbf{r}} - \vec{\mathbf{r}}_0}{t} = \frac{2.0\,\text{m}\hat{\mathbf{i}} - 2.0\,\text{m}\hat{\mathbf{j}}}{2.0\,\text{s}} = \boxed{1.0\,\text{m/s}\left(\hat{\mathbf{i}} - \hat{\mathbf{j}}\right)}$$

(*c*) The velocity at the end of the 2.0 seconds is pointing in the $-\hat{\mathbf{j}}$ direction.

$$\vec{\mathbf{a}}_{\text{avg}} = \frac{\vec{\mathbf{v}} - \vec{\mathbf{v}}_0}{t} = \frac{-\left(\pi\,\text{m/s}\right)\hat{\mathbf{j}}}{2.0\,\text{s}} = \boxed{\left(-\pi/2\,\text{m/s}^2\right)\hat{\mathbf{j}}}$$

80. Since mass m is dangling, the tension in the cord must be equal to the weight of mass m, and so $F_T = mg$. That same tension is in the other end of the cord, maintaining the circular motion of mass M, and so $F_T = F_R = Ma_R = M v^2/r$. Equate the expressions for tension and solve for the velocity.

$$M v^2/r = mg \quad \rightarrow \quad v = \boxed{\sqrt{mgR/M}}$$

94. An object at the Earth's equator is rotating in a circle with a radius equal to the radius of the Earth, and a period equal to one day. Use that data to find the centripetal acceleration and then compare it to g.

$$a_R = \frac{v^2}{r} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2} \quad \rightarrow \quad \frac{a_R}{g} = \frac{\frac{4\pi^2\left(6.38\times10^6\,\text{m}\right)}{\left(86,400\,\text{s}\right)^2}}{\left(9.80\,\text{m/s}^2\right)} = 0.00344 \approx \boxed{\frac{3}{1000}}$$

So, for example, if we were to calculate the normal force on an object at the Earth's equator, we could not say $\sum F = F_N - mg = 0$. Instead, we would have the following.

$$\sum F = F_N - mg = -m\frac{v^2}{r} \quad \rightarrow \quad F_N = mg - m\frac{v^2}{r}$$

If we then assumed that $F_N = mg_{\text{eff}} = mg - m\frac{v^2}{r}$, then we see that the effective value of g is

$$g_{\text{eff}} = g - \frac{v^2}{r} = g - 0.003g = 0.997g.$$