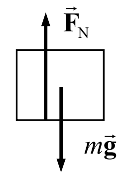
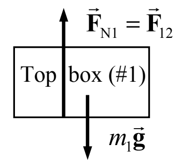
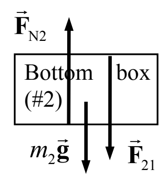
UCSD Phys 4A – Intro Mechanics – Winter 2016
Ch 4 Solutions

10. (*a*) The 20.0 kg box resting on the table has the free-body diagram shown. Its weight is $mg = (20.0\text{ kg})(9.80\text{ m/s}^2) = \boxed{196\text{ N}}$. Since the box is at rest, the net force on the box must be 0, and so the normal force must also be $\boxed{196\text{ N}}$.



(*b*) Free-body diagrams are shown for both boxes. $\vec{\mathbf{F}}_{12}$ is the force on box 1 (the top box) due to box 2 (the bottom box), and is the normal force on box 1. $\vec{\mathbf{F}}_{21}$ is the force on box 2 due to box 1, and has the same magnitude as $\vec{\mathbf{F}}_{12}$ by Newton's third law. $\vec{\mathbf{F}}_{\text{N2}}$ is the force of the table on box 2. That is the normal force on box 2. Since both boxes are at rest, the net force on each box must be 0. Write Newton's second law in the vertical direction for each box, taking the upward direction to be positive.





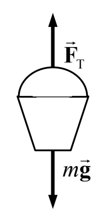
$$\sum F_1 = F_{\text{N1}} - m_1 g = 0$$

$$F_{\text{N1}} = m_1 g = (10.0\,\text{kg})(9.80\,\text{m/s}^2) = \boxed{98.0\text{ N}} = F_{12} = F_{21}$$

$$\sum F_2 = F_{\text{N2}} - F_{21} - m_2 g = 0$$

$$F_{\text{N2}} = F_{21} + m_2 g = 98.0\text{ N} + (20.0\,\text{kg})(9.80\,\text{m/s}^2) = \boxed{294\text{ N}}$$

$\boxed{13.}$ Choose up to be the positive direction. Write Newton's second law for the vertical direction, and solve for the acceleration.



$$\sum F = F_{\text{T}} - mg = ma$$

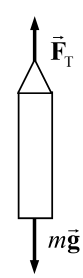
$$a = \frac{F_{\text{T}} - mg}{m} = \frac{163\,\text{N} - (14.0\,\text{kg})(9.80\,\text{m/s}^2)}{14.0\,\text{kg}} = \boxed{1.8\,\text{m/s}^2}$$

Since the acceleration is positive, the bucket has an $\boxed{\text{upward}}$ acceleration.

21. (*a*) Since the rocket is exerting a downward force on the gases, the gases will exert an upward force on the rocket, typically called the thrust. The free-body diagram for the rocket shows two forces – the thrust and the weight. Newton's second law can be used to find the acceleration of the rocket.

$$\sum F = F_{\text{T}} - mg = ma \quad \rightarrow$$

$$a = \frac{F_{\text{T}} - mg}{m} = \frac{3.55\times10^7\,\text{N} - (2.75\times10^6\,\text{kg})(9.80\,\text{m/s}^2)}{(2.75\times10^6\,\text{kg})} = 3.109\,\text{m/s}^2 \approx \boxed{3.1\,\text{m/s}^2}$$



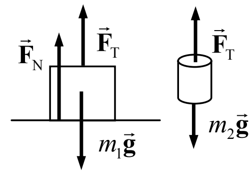
(*b*) The velocity can be found from Eq. 2-12a.

$$v = v_0 + at = 0 + (3.109\,\text{m/s}^2)(8.0\,\text{s}) = 24.872\,\text{m/s} \approx \boxed{25\,\text{m/s}}$$

(*c*) The time to reach a displacement of 9500 m can be found from Eq. 2-12b.

$$x - x_0 = v_0 t + \tfrac{1}{2}at^2 \quad \rightarrow \quad t = \sqrt{\frac{2(x - x_0)}{a}} = \sqrt{\frac{2(9500\,\text{m})}{(3.109\,\text{m/s}^2)}} = \boxed{78\,\text{s}}$$

27. Free-body diagrams for the box and the weight are shown below. The tension exerts the same magnitude of force on both objects.



(*a*) If the weight of the hanging weight is less than the weight of the box, the objects will not move, and the tension will be the same as the weight of the hanging weight. The acceleration of the box will also be zero, and so the sum of the forces on it will be zero. For the box,

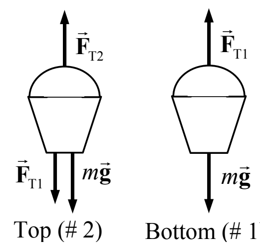
$$F_N + F_T - m_1 g = 0 \;\rightarrow\; F_N = m_1 g - F_T = m_1 g - m_2 g = 77.0\text{N} - 30.0\text{ N} = \boxed{47.0\text{ N}}$$

(*b*) The same analysis as for part (*a*) applies here.

$$F_N = m_1 g - m_2 g = 77.0\text{ N} - 60.0\text{ N} = \boxed{17.0\text{ N}}$$

(*c*) Since the hanging weight has more weight than the box on the table, the box on the table will be lifted up off the table, and normal force of the table on the box will be $\boxed{0\text{ N}}$.

33. We draw free-body diagrams for each bucket.



(*a*) Since the buckets are at rest, their acceleration is 0. Write Newton's second law for each bucket, calling UP the positive direction.

$$\sum F_1 = F_{T1} - mg = 0 \;\rightarrow$$

$$F_{T1} = mg = (3.2\,\text{kg})(9.80\,\text{m/s}^2) = \boxed{31\,\text{N}}$$

$$\sum F_2 = F_{T2} - F_{T1} - mg = 0 \;\rightarrow$$

$$F_{T2} = F_{T1} + mg = 2mg = 2(3.2\,\text{kg})(9.80\,\text{m/s}^2) = \boxed{63\,\text{N}}$$

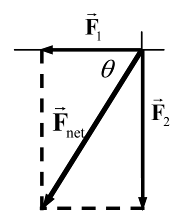
(*b*) Now repeat the analysis, but with a non-zero acceleration. The free-body diagrams are unchanged.

$$\sum F_1 = F_{T1} - mg = ma \;\rightarrow$$

$$F_{T1} = mg + ma = (3.2\,\text{kg})(9.80\,\text{m/s}^2 + 1.25\,\text{m/s}^2) = 35.36\,\text{N} \approx \boxed{35\,\text{N}}$$

$$\sum F_2 = F_{T2} - F_{T1} - mg = ma \;\rightarrow\; F_{T2} = F_{T1} + mg + ma = 2F_{T1} = \boxed{71\,\text{N}}$$

$\boxed{37.}$ The net force in each case is found by vector addition with components.



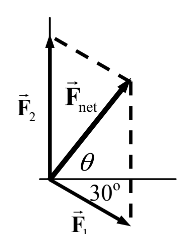
(*a*) $F_{\text{Net x}} = -F_1 = -10.2\text{ N} \quad F_{\text{Net y}} = -F_2 = -16.0\text{ N}$

$$F_{\text{Net}} = \sqrt{(-10.2)^2 + (-16.0)^2} = 19.0\text{ N} \qquad \theta = \tan^{-1}\frac{-16.0}{-10.2} = 57.48°$$

The actual angle from the *x*-axis is then $237.48°$. Thus the net force is

$$F_{\text{Net}} = \boxed{19.0\text{ N at }237.5°}$$

$$a = \frac{F_{\text{Net}}}{m} = \frac{19.0\,\text{N}}{18.5\,\text{kg}} = \boxed{1.03\,\text{m/s}^2\text{ at }237.5°}$$

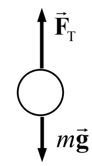


(*b*) $F_{\text{Net x}} = F_1\cos 30° = 8.833\text{ N} \quad F_{\text{Net y}} = F_2 - F_1\sin 30° = 10.9\,\text{N}$

$$F_{\text{Net}} = \sqrt{(8.833\text{ N})^2 + (10.9\text{ N})^2} = 14.03\text{N} \approx \boxed{14.0\text{ N}}$$

$$\theta = \tan^{-1}\frac{10.9}{8.833} = \boxed{51.0°} \qquad a = \frac{F_{\text{Net}}}{m} = \frac{14.03\text{ N}}{18.5\text{ kg}} = \boxed{0.758\,\text{m/s}^2}\text{ at }\boxed{51.0°}$$

44. For each object, we have the free-body diagram shown, assuming that the string doesn't break. Newton's second law is used to get an expression for the tension. Since the string broke for the 2.10 kg mass, we know that the required tension to accelerate that mass was more than 22.2 N. Likewise, since the string didn't break for the 2.05 kg mass, we know that the required tension to accelerate that mass was less than 22.2 N. These relationships can be used to get the range of accelerations.



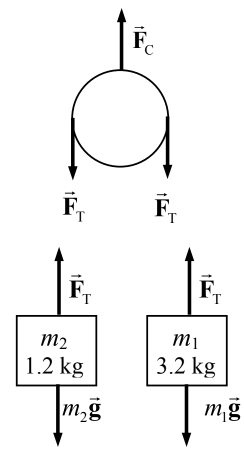
$$\sum F = F_T - mg = ma \quad \rightarrow \quad F_T = m(a+g)$$

$$F_{T_{max}} < m_{2.10}(a+g) \;\; ; \;\; F_{T_{max}} > m_{2.05}(a+g) \quad \rightarrow \quad \frac{F_{T_{max}}}{m_{2.10}} - g < a \;\; ; \;\; \frac{F_{T_{max}}}{m_{2.05}} - g > a \quad \rightarrow$$

$$\frac{F_{T_{max}}}{m_{2.10}} - g < a < \frac{F_{T_{max}}}{m_{2.05}} - g \quad \rightarrow \quad \frac{22.2\,\text{N}}{2.10\,\text{kg}} - 9.80\,\text{m/s}^2 < a < \frac{22.2\,\text{N}}{2.05\,\text{kg}} - 9.80\,\text{m/s}^2 \quad \rightarrow$$

$$0.77\,\text{m/s}^2 < a < 1.03\,\text{m/s}^2 \quad \rightarrow \quad \boxed{0.8\,\text{m/s}^2 < a < 1.0\,\text{m/s}^2}$$

54. We draw a free-body diagram for each mass. We choose UP to be the positive direction. The tension force in the cord is found from analyzing the two hanging masses. Notice that the same tension force is applied to each mass. Write Newton's second law for each of the masses.



$$F_T - m_1 g = m_1 a_1 \qquad F_T - m_2 g = m_2 a_2$$

Since the masses are joined together by the cord, their accelerations will have the same magnitude but opposite directions. Thus $a_1 = -a_2$. Substitute this into the force expressions and solve for the tension force.

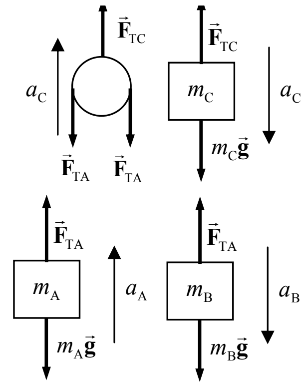
$$F_T - m_1 g = -m_1 a_2 \quad \rightarrow \quad F_T = m_1 g - m_1 a_2 \quad \rightarrow \quad a_2 = \frac{m_1 g - F_T}{m_1}$$

$$F_T - m_2 g = m_2 a_2 = m_2\left(\frac{m_1 g - F_T}{m_1}\right) \quad \rightarrow \quad F_T = \frac{2 m_1 m_2 g}{m_1 + m_2}$$

Apply Newton's second law to the stationary pulley.

$$F_C - 2F_T = 0 \quad \rightarrow \quad F_C = 2F_T = \frac{4 m_1 m_2 g}{m_1 + m_2} = \frac{4(3.2\,\text{kg})(1.2\,\text{kg})(9.80\,\text{m/s}^2)}{4.4\,\text{kg}} = \boxed{34\,\text{N}}$$

56. Because the pulleys are massless, the net force on them must be 0. Because the cords are massless, the tension will be the same at both ends of the cords. Use the free-body diagrams to write Newton's second law for each mass. We are using the same approach taken in problem 47, where we take the direction of acceleration to be positive in the direction of motion of the object. We assume that m_C is falling, m_B is falling relative to its pulley, and m_A is rising relative to its pulley. Also note that if the acceleration of m_A relative to the pulley above it is a_R, then $a_A = a_R + a_C$. Then, the acceleration of m_B is $a_B = a_R - a_C$, since a_C is in the opposite direction of a_B.



$$m_A: \quad \sum F = F_{TA} - m_A g = m_A a_A = m_A\left(a_R + a_C\right)$$
$$m_B: \quad \sum F = m_B g - F_{TA} = m_B a_B = m_B\left(a_R - a_C\right)$$
$$m_C: \quad \sum F = m_C g - F_{TC} = m_C a_C$$
$$\text{pulley:} \quad \sum F = F_{TC} - 2F_{TA} = 0 \quad \rightarrow \quad F_{TC} = 2F_{TA}$$

Re-write this system as three equations in three unknowns F_{TA}, a_R, a_C.

$$F_{TA} - m_A g = m_A\left(a_R + a_C\right) \quad \rightarrow \quad F_{TA} - m_A a_C - m_A a_R = m_A g$$
$$m_B g - F_{TA} = m_B\left(a_R - a_C\right) \quad \rightarrow \quad F_{TA} - m_B a_C + m_B a_R = m_B g$$
$$m_C g - 2F_{TA} = m_C a_C \quad \rightarrow \quad 2F_{TA} + m_C a_C = m_C g$$

This system now needs to be solved. One method to solve a system of linear equations is by determinants. We show that for a_C.

$$a_C = \frac{\begin{vmatrix} 1 & m_A & -m_A \\ 1 & m_B & m_B \\ 2 & m_C & 0 \end{vmatrix}}{\begin{vmatrix} 1 & -m_A & -m_A \\ 1 & -m_B & m_B \\ 2 & m_C & 0 \end{vmatrix}} g = \frac{-m_B m_C + m_A\left(2m_B\right) - m_A\left(m_C - 2m_B\right)}{-m_B m_C - m_A\left(2m_B\right) - m_A\left(m_C + 2m_B\right)} g$$

$$= \frac{4m_A m_B - m_A m_C - m_B m_C}{-4m_A m_B - m_A m_C - m_B m_C} g = \frac{m_A m_C + m_B m_C - 4m_A m_B}{4m_A m_B + m_A m_C + m_B m_C} g$$

Similar manipulations give the following results.

$$a_R = \frac{2(m_A m_C - m_B m_C)}{4m_A m_B + m_A m_C + m_B m_C} g \quad ; \quad F_{TA} = \frac{4m_A m_B m_C}{4m_A m_B + m_A m_C + m_B m_C} g$$

(*a*) The accelerations of the three masses are found below.

$$a_A = a_R + a_C = \frac{2(m_A m_C - m_B m_C)}{4m_A m_B + m_A m_C + m_B m_C} g + \frac{m_A m_C + m_B m_C - 4m_A m_B}{4m_A m_B + m_A m_C + m_B m_C} g$$

$$= \boxed{\frac{3m_A m_C - m_B m_C - 4m_A m_B}{4m_A m_B + m_A m_C + m_B m_C} g}$$

$$a_B = a_R - a_C = \frac{2(m_A m_C - m_B m_C)}{4m_A m_B + m_A m_C + m_B m_C} g - \frac{m_A m_C + m_B m_C - 4m_A m_B}{4m_A m_B + m_A m_C + m_B m_C} g$$

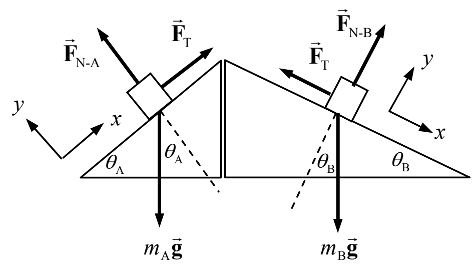
$$= \boxed{\frac{m_A m_C - 3m_B m_C + 4m_A m_B}{4m_A m_B + m_A m_C + m_B m_C} g}$$

$$a_C = \boxed{\frac{m_A m_C + m_B m_C - 4m_A m_B}{4m_A m_B + m_A m_C + m_B m_C} g}$$

(*b*) The tensions are shown below.

$$F_{TA} = \boxed{\frac{4m_A m_B m_C}{4m_A m_B + m_A m_C + m_B m_C} g} \quad ; \quad F_{TC} = 2F_{TA} = \boxed{\frac{8m_A m_B m_C}{4m_A m_B + m_A m_C + m_B m_C} g}$$

69. (*a*) A free-body diagram is shown for each block. We define the positive *x*-direction for m_A to be up its incline, and the positive *x*-direction for m_B to be down its incline. With that definition the masses will both have the same acceleration. Write Newton's second law for each body in the *x* direction, and combine those equations to find the acceleration.



$$m_A: \quad \sum F_x = F_T - m_A g \sin\theta_A = m_A a$$

$$m_B: \quad \sum F_x = m_B g \sin\theta_B - F_T = m_B a \qquad \text{add these two equations}$$

$$(F_T - m_A g \sin\theta_A) + (m_B g \sin\theta_B - F_T) = m_A a + m_B a \;\rightarrow\; a = \boxed{\frac{m_B \sin\theta_B - m_A \sin\theta_A}{m_A + m_B} g}$$

(*b*) For the system to be at rest, the acceleration must be 0.

$$a = \frac{m_B \sin\theta_B - m_A \sin\theta_A}{m_A + m_B} g = 0 \;\rightarrow\; m_B \sin\theta_B - m_A \sin\theta_A \;\rightarrow$$

$$m_B = m_A \frac{\sin\theta_A}{\sin\theta_B} = (5.0\,\text{kg})\frac{\sin 32^\circ}{\sin 23^\circ} = \boxed{6.8\,\text{kg}}$$

The tension can be found from one of the Newton's second law expression from part (*a*).

$$m_A: \quad F_T - m_A g \sin\theta_A = 0 \;\rightarrow\; F_T = m_A g \sin\theta_A = (5.0\,\text{kg})(9.80\,\text{m/s}^2)\sin 32^\circ = \boxed{26\,\text{N}}$$

(*c*) As in part (*b*), the acceleration will be 0 for constant velocity in either direction.

$$a = \frac{m_B \sin\theta_B - m_A \sin\theta_A}{m_A + m_B} g = 0 \;\rightarrow\; m_B \sin\theta_B - m_A \sin\theta_A \;\rightarrow$$

$$\frac{m_A}{m_B} = \frac{\sin\theta_B}{\sin\theta_A} = \frac{\sin 23^\circ}{\sin 32^\circ} = \boxed{0.74}$$

$\boxed{79.}$ (*a*) We assume that the maximum horizontal force occurs when the train is moving very slowly, and so the air resistance is negligible. Thus the maximum acceleration is given by the following.

$$a_{max} = \frac{F_{max}}{m} = \frac{4\times10^5\,\mathrm{N}}{6.4\times10^5\,\mathrm{kg}} = 0.625\,\mathrm{m/s^2} \approx \boxed{0.6\,\mathrm{m/s^2}}$$

(*b*) At top speed, we assume that the train is moving at constant velocity. Therefore the net force on the train is 0, and so the air resistance and friction forces together must be of the same magnitude as the horizontal pushing force, which is $\boxed{1.5\times10^5\,\mathrm{N}}$.