PHYS 4A – Mechanics by Prof. Hans Paar – Winter 2016

Homework 2 Solutions

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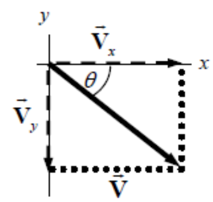
Chapter 3 problems

3)

Given that $V_x = 7.80$ units and $V_y = -6.40$ units, the magnitude of $\vec{\mathbf{V}}$ is given by $V = \sqrt{V_x^2 + V_y^2} = \sqrt{7.80^2 + (-6.40)^2} = \boxed{10.1\text{ units}}$. The direction is given by $\theta = \tan^{-1}\dfrac{-6.40}{7.80} = \boxed{-39.4°}$, 39.4° below the positive x-axis.



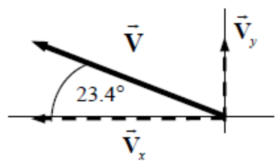
5)

(a) See the accompanying diagram

(b) $V_x = -24.8\cos 23.4° = \boxed{-22.8\text{ units}}$ $\quad V_y = 24.8\sin 23.4° = \boxed{9.85\text{ units}}$

(c) $V = \sqrt{V_x^2 + V_y^2} = \sqrt{(-22.8)^2 + (9.85)^2} = \boxed{24.8\text{ units}}$

$\theta = \tan^{-1}\dfrac{9.85}{22.8} = \boxed{23.4°\text{above the } -x \text{ axis}}$



6)

We see from the diagram that $\vec{\mathbf{A}} = 6.8\hat{\mathbf{i}}$ and $\vec{\mathbf{B}} = -5.5\hat{\mathbf{i}}$.

(a) $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} = 6.8\hat{\mathbf{i}} + (-5.5)\hat{\mathbf{i}} = \boxed{1.3\hat{\mathbf{i}}}$. The magnitude is $\boxed{1.3\text{ units}}$, and the direction is $\boxed{+x}$.

(b) $\vec{\mathbf{C}} = \vec{\mathbf{A}} - \vec{\mathbf{B}} = 6.8\hat{\mathbf{i}} - (-5.5)\hat{\mathbf{i}} = \boxed{12.3\hat{\mathbf{i}}}$. The magnitude is $\boxed{12.3\text{ units}}$, and the direction is $\boxed{+x}$.

(c) $\vec{\mathbf{C}} = \vec{\mathbf{B}} - \vec{\mathbf{A}} = (-5.5)\hat{\mathbf{i}} - 6.8\hat{\mathbf{i}} = \boxed{-12.3\hat{\mathbf{i}}}$. The magnitude is $\boxed{12.3\text{ units}}$, and the direction is $\boxed{-x}$.

8)

(a) $\vec{\mathbf{V}}_1 = -6.0\hat{\mathbf{i}} + 8.0\hat{\mathbf{j}} \quad V_1 = \sqrt{6.0^2 + 8.0^2} = \boxed{10.0} \quad \theta = \tan^{-1}\dfrac{8.0}{-6.0} = \boxed{127°}$

(b) $\vec{\mathbf{V}}_2 = 4.5\hat{\mathbf{i}} - 5.0\hat{\mathbf{j}} \quad V_2 = \sqrt{4.5^2 + 5.0^2} = \boxed{6.7} \quad \theta = \tan^{-1}\dfrac{-5.0}{4.5} = \boxed{312°}$

(c) $\vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2 = (-6.0\hat{\mathbf{i}} + 8.0\hat{\mathbf{j}}) + (4.5\hat{\mathbf{i}} - 5.0\hat{\mathbf{j}}) = -1.5\hat{\mathbf{i}} + 3.0\hat{\mathbf{j}}$

$\left|\vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2\right| = \sqrt{1.5^2 + 3.0^2} = \boxed{3.4} \quad \theta = \tan^{-1}\dfrac{3.0}{-1.5} = \boxed{117°}$

(d) $\vec{\mathbf{V}}_2 - \vec{\mathbf{V}}_1 = (4.5\hat{\mathbf{i}} - 5.0\hat{\mathbf{j}}) - (-6.0\hat{\mathbf{i}} + 8.0\hat{\mathbf{j}}) = 10.5\hat{\mathbf{i}} - 13.0\hat{\mathbf{j}}$

$\left|\vec{\mathbf{V}}_2 - \vec{\mathbf{V}}_1\right| = \sqrt{10.5^2 + 13.0^2} = \boxed{16.7} \quad \theta = \tan^{-1}\dfrac{-13.0}{10.5} = \boxed{309°}$

10)

$A_x = 44.0\cos 28.0° = 38.85 \qquad A_y = 44.0\sin 28.0° = 20.66$

$B_x = -26.5\cos 56.0° = -14.82 \qquad B_y = 26.5\sin 56.0° = 21.97$

$C_x = 31.0\cos 270° = 0.0 \qquad C_y = 31.0\sin 270° = -31.0$

(a) $\left(\vec{\mathbf{A}}+\vec{\mathbf{B}}+\vec{\mathbf{C}}\right)_x = 38.85+(-14.82)+0.0 = 24.03 = \boxed{24.0}$

$\left(\vec{\mathbf{A}}+\vec{\mathbf{B}}+\vec{\mathbf{C}}\right)_y = 20.66+21.97+(-31.0) = 11.63 = \boxed{11.6}$

(b) $\left|\vec{\mathbf{A}}+\vec{\mathbf{B}}+\vec{\mathbf{C}}\right| = \sqrt{(24.03)^2+(11.63)^2} = \boxed{26.7} \qquad \theta = \tan^{-1}\frac{11.63}{24.03} = \boxed{25.8°}$

13)

$A_x = 44.0\cos 28.0° = 38.85 \qquad A_y = 44.0\sin 28.0° = 20.66$

$B_x = -26.5\cos 56.0° = -14.82 \qquad B_y = 26.5\sin 56.0° = 21.97$

$C_x = 31.0\cos 270° = 0.0 \qquad C_y = 31.0\sin 270° = -31.0$

(a) $\left(\vec{\mathbf{B}}-2\vec{\mathbf{A}}\right)_x = -14.82-2(38.85) = -92.52 \qquad \left(\vec{\mathbf{B}}-2\vec{\mathbf{A}}\right)_y = 21.97-2(20.66) = -19.35$

Note that since both components are negative, the vector is in the 3rd quadrant.

$\vec{\mathbf{B}}-2\vec{\mathbf{A}} = \boxed{-92.5\hat{\mathbf{i}}-19.4\hat{\mathbf{j}}}$

$\left|\vec{\mathbf{B}}-2\vec{\mathbf{A}}\right| = \sqrt{(-92.52)^2+(-19.35)^2} = \boxed{94.5} \qquad \theta = \tan^{-1}\frac{-19.35}{-92.52} = \boxed{11.8°\text{below} -x \text{ axis}}$

(b) $\left(2\vec{\mathbf{A}}-3\vec{\mathbf{B}}+2\vec{\mathbf{C}}\right)_x = 2(38.85)-3(-14.82)+2(0.0) = 122.16$

$\left(2\vec{\mathbf{A}}-3\vec{\mathbf{B}}+2\vec{\mathbf{C}}\right)_y = 2(20.66)-3(21.97)+2(-31.0) = -86.59$

Note that since the x component is positive and the y component is negative, the vector is in the 4th quadrant.

$2\vec{\mathbf{A}}-3\vec{\mathbf{B}}+2\vec{\mathbf{C}} = \boxed{122\hat{\mathbf{i}}-86.6\hat{\mathbf{j}}}$

$\left|2\vec{\mathbf{A}}-3\vec{\mathbf{B}}+2\vec{\mathbf{C}}\right| = \sqrt{(122.16)^2+(-86.59)^2} = \boxed{150} \qquad \theta = \tan^{-1}\frac{-86.59}{122.16} = \boxed{35.3°\text{below} +x \text{ axis}}$

17)

Differentiate the position vector in order to determine the velocity, and differentiate the velocity in order to determine the acceleration.

$\vec{\mathbf{r}} = \left(9.60t\,\hat{\mathbf{i}}+8.85\,\hat{\mathbf{j}}-1.00t^2\hat{\mathbf{k}}\right)\text{m} \;\rightarrow\; \vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} = \boxed{\left(9.60\,\hat{\mathbf{i}}-2.00t\,\hat{\mathbf{k}}\right)\text{m/s}} \;\rightarrow$

$\vec{\mathbf{a}} = \frac{d\vec{\mathbf{v}}}{dt} = \boxed{-2.00\,\hat{\mathbf{k}}\,\text{m/s}^2}$

18)

The average velocity is found from the displacement at the two times.

$$\vec{\mathbf{v}}_{\text{avg}} = \frac{\vec{\mathbf{r}}(t_2)-\vec{\mathbf{r}}(t_1)}{t_2-t_1}$$

$$= \frac{\left[\left(9.60(3.00)\,\hat{\mathbf{i}}+8.85\,\hat{\mathbf{j}}-(3.00)^2\,\hat{\mathbf{k}}\right)\text{m}\right]-\left[\left(9.60(1.00)\,\hat{\mathbf{i}}+8.85\,\hat{\mathbf{j}}-(1.00)^2\,\hat{\mathbf{k}}\right)\text{m}\right]}{2.00\,\text{s}}$$

$$= \boxed{\left(9.60\,\hat{\mathbf{i}}-4.00\,\hat{\mathbf{k}}\right)\text{m/s}}$$

The magnitude of the instantaneous velocity is found from the velocity vector.

$$\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} = \boxed{\left(9.60\,\hat{\mathbf{i}} - 2.00t\,\hat{\mathbf{k}}\right)\text{m/s}}$$

$$\vec{\mathbf{v}}(2.00) = \left(9.60\,\hat{\mathbf{i}} - (2.00)(2.00)\,\hat{\mathbf{k}}\right)\text{m/s} = \left(9.60\,\hat{\mathbf{i}} - 4.00\,\hat{\mathbf{k}}\right)\text{m/s} \quad \rightarrow$$

$$v = \sqrt{(9.60)^2 + (4.00)^2}\ \text{m/s} = \boxed{10.4\,\text{m/s}}$$

Note that, since the acceleration of this object is constant, the average velocity over the time interval is equal to the instantaneous velocity at the midpoint of the time interval.

19)

From the original position vector, we have $x = 9.60t$, $y = 8.85$, $z = -1.00t^2$. Thus

$z = -\left(\dfrac{x}{9.60}\right)^2 = -ax^2$, $y = 8.85$. This is the equation for a $\boxed{\text{parabola}}$ in the x-z plane that has its vertex at coordinate (0,8.85,0) and opens downward.

24)

Since the acceleration vector is constant, Eqs. 3-13a and 3-13b are applicable. The particle reaches its maximum x coordinate when the x velocity is 0. Note that $\vec{\mathbf{v}}_0 = 5.0\,\text{m/s}\,\hat{\mathbf{i}}$ and $\vec{\mathbf{r}}_0 = 0$.

$$\vec{\mathbf{v}} = \vec{\mathbf{v}}_0 + \vec{\mathbf{a}}t = 5.0\,\hat{\mathbf{i}}\,\text{m/s} + \left(-3.0t\,\hat{\mathbf{i}} + 4.5t\,\hat{\mathbf{j}}\right)\text{m/s}$$

$$v_x = (5.0 - 3.0t)\,\text{m/s} \quad \rightarrow \quad v_x = 0 = (5.0 - 3.0t_{x\text{-max}})\,\text{m/s} \quad \rightarrow \quad t_{x\text{-max}} = \frac{5.0\,\text{m/s}}{3.0\,\text{m/s}^2} = 1.67\,\text{s}$$

$$\vec{\mathbf{v}}(t_{x\text{-max}}) = 5.0\,\hat{\mathbf{i}}\,\text{m/s} + \left[-3.0(1.67)\,\hat{\mathbf{i}} + 4.5(1.67)t\,\hat{\mathbf{j}}\right]\text{m/s} = \boxed{7.5\,\text{m/s}\,\hat{\mathbf{j}}}$$

$$\vec{\mathbf{r}} = \vec{\mathbf{r}}_0 + \vec{\mathbf{v}}_0 t + \tfrac{1}{2}\vec{\mathbf{a}}t^2 = 5.0t\,\hat{\mathbf{i}}\,\text{m} + \tfrac{1}{2}\left(-3.0t^2\,\hat{\mathbf{i}} + 4.5t^2\,\hat{\mathbf{j}}\right)\text{m}$$

$$\vec{\mathbf{r}}(t_{x\text{-max}}) = 5.0(1.67)\,\hat{\mathbf{i}}\,\text{m/s} + \tfrac{1}{2}\left[-3.0(1.67)^2\,\hat{\mathbf{i}} + 4.5(1.67)^2\,\hat{\mathbf{j}}\right]\text{m} = \boxed{4.2\,\hat{\mathbf{i}}\,\text{m} + 6.3\,\hat{\mathbf{j}}\,\text{m}}$$

28)

Choose downward to be the positive y direction. The origin will be at the point where the tiger leaps from the rock. In the horizontal direction, $v_{x0} = 3.2\,\text{m/s}$ and $a_x = 0$. In the vertical direction, $v_{y0} = 0$, $a_y = 9.80\,\text{m/s}^2$, $y_0 = 0$, and the final location $y = 7.5\,\text{m}$. The time for the tiger to reach the ground is found from applying Eq. 2-12b to the vertical motion.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 7.5\text{m} = 0 + 0 + \tfrac{1}{2}\left(9.80\,\text{m/s}^2\right)t^2 \quad \rightarrow \quad t = \sqrt{\frac{2(7.5\text{m})}{9.80\,\text{m/s}^2}} = 1.24\ \text{sec}$$

The horizontal displacement is calculated from the constant horizontal velocity.

$$\Delta x = v_x t = (3.2\,\text{m/s})(1.24\ \text{sec}) = \boxed{4.0\ \text{m}}$$

33)

Choose the point at which the football is kicked the origin, and choose upward to be the positive y direction. When the football reaches the ground again, the y displacement is 0. For the football, $v_{y0} = (18.0\sin 38.0°)\,\text{m/s}$, $a_y = -9.80\,\text{m/s}^2$, and the final y velocity will be the opposite of the starting y velocity. Use Eq. 2-12a to find the time of flight.

$$v_y = v_{y0} + at \quad \rightarrow \quad t = \frac{v_y - v_{y0}}{a} = \frac{(-18.0\sin 38.0°)\,\text{m/s} - (18.0\sin 38.0°)\,\text{m/s}}{-9.80\,\text{m/s}^2} = \boxed{2.26\,\text{s}}$$

36)

Choose the origin to be the point of launch, and upwards to be the positive y direction. The initial velocity of the projectile is v_0, the launching angle is θ_0, $a_y = -g$, $y_0 = 0$, and $v_{y0} = v_0 \sin\theta_0$. Eq. 2-12a is used to find the time required to reach the highest point, at which $v_y = 0$.

$$v_y = v_{y0} + at_{\text{up}} \quad \rightarrow \quad t_{\text{up}} = \frac{v_y - v_{y0}}{a} = \frac{0 - v_0 \sin\theta_0}{-g} = \frac{v_0 \sin\theta_0}{g}$$

Eq. 2-12c is used to find the height at this highest point.

$$v_y^2 = v_{y0}^2 + 2a_y\left(y_{\max} - y_0\right) \quad \rightarrow \quad y_{\max} = y_0 + \frac{v_y^2 - v_{y0}^2}{2a_y} = 0 + \frac{-v_0^2 \sin^2\theta_0}{-2g} = \frac{v_0^2 \sin^2\theta_0}{2g}$$

Eq. 2-12b is used to find the time for the object to fall the other part of the path, with a starting y velocity of 0 and a starting height of $y_0 = \dfrac{v_0^2 \sin^2\theta_0}{2g}$.

$$y = y_o + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 0 = \frac{v_0^2 \sin^2\theta_0}{2g} + 0t_{\text{down}} - \tfrac{1}{2}gt_{\text{down}}^2 \quad \rightarrow \quad t_{\text{down}} = \frac{v_0 \sin\theta_0}{g}$$

A comparison shows that $\boxed{t_{\text{up}} = t_{\text{down}}}$.

46)

Choose the origin to be at ground level, under the place where the projectile is launched, and upwards to be the positive y direction. For the projectile, $v_0 = 65.0\,\text{m/s}$, $\theta_0 = 35.0°$, $a_y = -g$, $y_0 = 115\,\text{m}$, and $v_{y0} = v_0 \sin\theta_0$.

(a) The time taken to reach the ground is found from Eq. 2-12b, with a final height of 0.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 0 = y_0 + v_0 \sin\theta_0 t - \tfrac{1}{2}gt^2 \quad \rightarrow$$

$$t = \frac{-v_0 \sin\theta_0 \pm \sqrt{v_0^2 \sin^2\theta_0 - 4\left(-\tfrac{1}{2}g\right)y_0}}{2\left(-\tfrac{1}{2}g\right)} = 9.964\,\text{s}\,,\; -2.3655\,\text{s} = \boxed{9.96\,\text{s}}$$

Choose the positive time since the projectile was launched at time $t = 0$.

(b) The horizontal range is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t = \left(v_0 \cos\theta_0\right)t = \left(65.0\,\text{m/s}\right)\left(\cos 35.0°\right)\left(9.964\,\text{s}\right) = \boxed{531\,\text{m}}$$

(c) At the instant just before the particle reaches the ground, the horizontal component of its velocity is the constant $v_x = v_0 \cos\theta_0 = \left(65.0\,\text{m/s}\right)\cos 35.0° = \boxed{53.2\,\text{m/s}}$. The vertical component is found from Eq. 2-12a.

$$v_y = v_{y0} + at = v_0 \sin\theta_0 - gt = \left(65.0\,\text{m/s}\right)\sin 35.0° - \left(9.80\,\text{m/s}^2\right)\left(9.964\,\text{s}\right)$$

$$= \boxed{-60.4\,\text{m/s}}$$

(d) The magnitude of the velocity is found from the x and y components calculated in part (c) above.

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{\left(53.2\,\text{m/s}\right)^2 + \left(-60.4\,\text{m/s}\right)^2} = \boxed{80.5\,\text{m/s}}$$

(e) The direction of the velocity is $\theta = \tan^{-1}\dfrac{v_y}{v_x} = \tan^{-1}\dfrac{-60.4}{53.2} = -48.6°$, and so the object is moving $\boxed{48.6° \text{ below the horizon}}$.

(f) The maximum height above the cliff top reached by the projectile will occur when the y-velocity is 0, and is found from Eq. 2-12c.

$$v_y^2 = v_{y0}^2 + 2a_y\left(y - y_0\right) \quad \rightarrow \quad 0 = v_0^2 \sin^2\theta_0 - 2gy_{\max}$$

$$y_{\max} = \frac{v_0^2 \sin^2\theta_0}{2g} = \frac{\left(65.0\,\text{m/s}\right)^2 \sin^2 35.0°}{2\left(9.80\,\text{m/s}^2\right)} = \boxed{70.9\,\text{m}}$$

55)

Choose the origin to be at the bottom of the hill, just where the incline starts. The equation of the line describing the hill is $y_2 = x\tan\phi$. The equations of the motion of the object are $y_1 = v_{0y}t + \frac{1}{2}a_y t^2$ and $x = v_{0x}t$, with $v_{0x} = v_0\cos\theta$ and $v_{0y} = v_0\sin\theta$. Solve the horizontal equation for the time of flight, and insert that into the vertical projectile motion equation.

$$t = \frac{x}{v_{0x}} = \frac{x}{v_0\cos\theta} \quad\rightarrow\quad y_1 = v_0\sin\theta\frac{x}{v_0\cos\theta} - \tfrac{1}{2}g\left(\frac{x}{v_0\cos\theta}\right)^2 = x\tan\theta - \frac{gx^2}{2v_0^2\cos^2\theta}$$

Equate the y-expressions for the line and the parabola to find the location where the two x-coordinates intersect.

$$x\tan\phi = x\tan\theta - \frac{gx^2}{2v_0^2\cos^2\theta} \quad\rightarrow\quad \tan\theta - \tan\phi = \frac{gx}{2v_0^2\cos^2\theta} \quad\rightarrow$$

$$x = \frac{(\tan\theta - \tan\phi)}{g}2v_0^2\cos^2\theta$$

This intersection x-coordinate is related to the desired quantity d by $x = d\cos\phi$.

$$d\cos\phi = (\tan\theta - \tan\phi)\frac{2v_0^2\cos^2\theta}{g} \quad\rightarrow\quad d = \frac{2v_0^2}{g\cos\phi}\left(\sin\theta\cos\theta - \tan\phi\cos^2\theta\right)$$

To maximize the distance, set the derivative of d with respect to θ equal to 0, and solve for θ.

$$\frac{d(d)}{d\theta} = \frac{2v_0^2}{g\cos\phi}\frac{d}{d\theta}\left(\sin\theta\cos\theta - \tan\phi\cos^2\theta\right)$$

$$= \frac{2v_0^2}{g\cos\phi}\left[\sin\theta(-\sin\theta) + \cos\theta(\cos\theta) - \tan\phi(2)\cos\theta(-\sin\theta)\right]$$

$$= \frac{2v_0^2}{g\cos\phi}\left[-\sin^2\theta + \cos^2\theta + 2\tan\phi\cos\theta\sin\theta\right] = \frac{2v_0^2}{g\cos\phi}\left[\cos 2\theta + \sin 2\theta\tan\phi\right] = 0$$

$$\cos 2\theta + \sin 2\theta\tan\phi = 0 \quad\rightarrow\quad \theta = \tfrac{1}{2}\tan^{-1}\left(-\frac{1}{\tan\phi}\right)$$

This expression can be confusing, because it would seem that a negative sign enters the solution. In order to get appropriate values, $180°$ or π radians must be added to the angle resulting from the inverse tangent operation, to have a positive angle. Thus a more appropriate expression would be the following:

$\theta = \tfrac{1}{2}\left[\pi + \tan^{-1}\left(-\frac{1}{\tan\phi}\right)\right]$. This can be shown to be equivalent to $\boxed{\theta = \frac{\phi}{2} + \frac{\pi}{4}}$, because

$$\tan^{-1}\left(-\frac{1}{\tan\phi}\right) = \tan^{-1}(-\cot\phi) = \cot^{-1}\cot\phi - \frac{\pi}{2} = \phi - \frac{\pi}{2}.$$