PHYS 4A – Mechanics by Prof. Hans Paar – Winter 2016

Homework 1 Solutions

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Chapter 1 problems

1)

$$(a) \quad 14 \text{ billion years} = \boxed{1.4\times10^{10}\text{ years}}$$

$$(b) \quad \left(1.4\times10^{10}\text{ y}\right)\left(3.156\times10^{7}\text{s}/1\text{ y}\right) = \boxed{4.4\times10^{17}\text{s}}$$

2)

$$(a) \quad 214 \quad \boxed{3 \text{ significant figures}}$$

$$(b) \quad 81.60 \quad \boxed{4 \text{ significant figures}}$$

$$(c) \quad 7.03 \quad \boxed{3 \text{ significant figures}}$$

$$(d) \quad 0.03 \quad \boxed{1 \text{ significant figure}}$$

$$(e) \quad 0.0086 \quad \boxed{2 \text{ significant figures}}$$

$$(f) \quad 3236 \quad \boxed{4 \text{ significant figures}}$$

$$(g) \quad 8700 \quad \boxed{2 \text{ significant figures}}$$

3)

$$(a) \quad 1.156 = \boxed{1.156\times10^{0}}$$

$$(b) \quad 21.8 = \boxed{2.18\times10^{1}}$$

$$(c) \quad 0.0068 = \boxed{6.8\times10^{-3}}$$

$$(d) \quad 328.65 = \boxed{3.2865\times10^{2}}$$

$$(e) \quad 0.219 = \boxed{2.19\times10^{-1}}$$

$$(f) \quad 444 = \boxed{4.44\times10^{2}}$$

4)

$$(a)\quad 8.69\times10^4 = \boxed{86,900}$$

$$(b)\quad 9.1\times10^3 = \boxed{9,100}$$

$$(c)\quad 8.8\times10^{-1} = \boxed{0.88}$$

$$(d)\quad 4.76\times10^2 = \boxed{476}$$

$$(e)\quad 3.62\times10^{-5} = \boxed{0.0000362}$$

8)

$\left(2.079\times10^2\,\text{m}\right)\left(0.082\times10^{-1}\right) = \boxed{1.7\,\text{m}}$. When multiplying, the result should have as many digits as the number with the least number of significant digits used in the calculation.

9)

θ (radians)	$\sin(\theta)$	$\tan(\theta)$
0	0.00	0.00
0.10	0.10	0.10
0.12	0.12	0.12
0.20	0.20	0.20
0.24	0.24	0.24
0.25	0.25	0.26

Keeping 2 significant figures in the angle, and expressing the angle in radians, the largest angle that has the same sine and tangent is $\boxed{0.24\text{ radians}}$. In degrees, the largest angle (keeping 2 significant figure) is 12°.

13)

Assuming a height of 5 feet 10 inches, then $5'10'' = (70\text{ in})(1\text{ m}/39.37\text{ in}) = \boxed{1.8\text{ m}}$. Assuming a weight of 165 lbs, then $(165\text{ lbs})(0.456\text{ kg}/1\text{ lb}) = \boxed{75.2\text{ kg}}$. Technically, pounds and mass measure two separate properties. To make this conversion, we have to assume that we are at a location where the acceleration due to gravity is 9.80 m/s^2.

Chapter 2 problems

2)

The average speed is given by Eq. 2-2.

$$\bar{v} = \Delta x/\Delta t = 235\ \text{km}/3.25\ \text{h} = \boxed{72.3\,\text{km/h}}$$

3)

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{8.5\,\text{cm} - 4.3\,\text{cm}}{4.5\,\text{s} - (-2.0\,\text{s})} = \frac{4.2\,\text{cm}}{6.5\,\text{s}} = \boxed{0.65\,\text{cm/s}}$$

The average speed cannot be calculated. To calculate the average speed, we would need to know the actual distance traveled, and it is not given. We only have the displacement.

6)

The time for the first part of the trip is calculated from the initial speed and the first distance.

$$\bar{v}_1 = \frac{\Delta x_1}{\Delta t_1} \rightarrow \Delta t_1 = \frac{\Delta x_1}{\bar{v}_1} = \frac{130\,\text{km}}{95\,\text{km/h}} = 1.37\,\text{h} = 82\,\text{min}$$

The time for the second part of the trip is now calculated.

$$\Delta t_2 = \Delta t_{\text{total}} - \Delta t_1 = 3.33\ \text{h} - 1.37\ \text{h} = 1.96\ \text{h} = 118\ \text{min}$$

The distance for the second part of the trip is calculated from the average speed for that part of the trip and the time for that part of the trip.

$$\bar{v}_2 = \frac{\Delta x_2}{\Delta t_2} \rightarrow \Delta x_2 = \bar{v}_2 \Delta t_2 = (65\,\text{km/h})(1.96\,\text{h}) = 127.5\ \text{km} = 1.3\times 10^2\,\text{km}$$

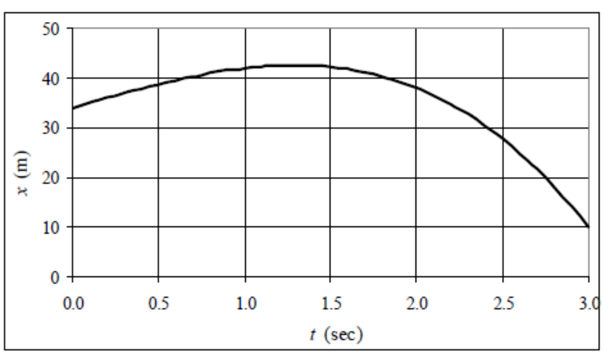
(*a*) The total distance is then $\Delta x_{\text{total}} = \Delta x_1 + \Delta x_2 = 130\ \text{km} + 127.5\ \text{km} = 257.5\ \text{km} \approx \boxed{2.6\times 10^2\,\text{km}}$.

(*b*) The average speed is NOT the average of the two speeds. Use the definition of average speed, Eq. 2-2.

$$\bar{v} = \frac{\Delta x_{\text{total}}}{\Delta t_{\text{total}}} = \frac{257.5\ \text{km}}{3.33\ \text{h}} = \boxed{77\,\text{km/h}}$$

8)

(*a*)



(*b*) The average velocity is the displacement divided by the elapsed time.

$$\bar{v} = \frac{x(3.0) - x(0.0)}{3.0\,\text{s} - 0.0\,\text{s}} = \frac{\left[34 + 10(3.0) - 2(3.0)^3\right]\text{m} - (34\,\text{m})}{3.0\,\text{s}} = \boxed{-8.0\,\text{m/s}}$$

(*c*) The instantaneous velocity is given by the derivative of the position function.

$$v = \frac{dx}{dt} = \left(10 - 6t^2\right)\text{m/s} \qquad 10 - 6t^2 = 0 \ \rightarrow\ t = \sqrt{\frac{5}{3}}\,\text{s} = \boxed{1.3\,\text{s}}$$

This can be seen from the graph as the "highest" point on the graph.

12)

Since the locomotives have the same speed, they each travel half the distance, 4.25 km. Find the time of travel from the average speed.

$$\bar{v} = \frac{\Delta x}{\Delta t} \rightarrow \Delta t = \frac{\Delta x}{\bar{v}} = \frac{4.25\text{ km}}{95\text{ km/h}} = 0.0447\text{ h}\left(\frac{60\text{ min}}{1\text{ h}}\right) = 2.68\text{ min} \approx \boxed{2.7\text{ min}}$$

18)

For the car to pass the train, the car must travel the length of the train AND the distance the train travels. The distance the car travels can thus be written as either $d_{car} = v_{car}t = (95\text{ km/h})t$ or $d_{car} = \ell_{train} + v_{train}t = 1.10\text{ km} + (75\text{ km/h})t$. To solve for the time, equate these two expressions for the distance the car travels.

$$(95\text{ km/h})t = 1.10\text{ km} + (75\text{ km/h})t \quad \rightarrow \quad t = \frac{1.10\text{ km}}{20\text{ km/h}} = 0.055\text{ h} = \boxed{3.3\text{ min}}$$

The distance the car travels during this time is $d = (95\text{ km/h})(0.055\text{ h}) = 5.225\text{ km} \approx \boxed{5.2\text{ km}}$.

If the train is traveling the opposite direction from the car, then the car must travel the length of the train MINUS the distance the train travels. Thus the distance the car travels can be written as either $d_{car} = (95\text{ km/h})t$ or $d_{car} = 1.10\text{ km} - (75\text{ km/h})t$. To solve for the time, equate these two expressions for the distance the car travels.

$$(95\text{ km/h})t = 1.10\text{ km} - (75\text{ km/h})t \quad \rightarrow \quad t = \frac{1.10\text{ km}}{170\text{ km/h}} = 6.47\times10^{-3}\text{ h} = \boxed{23.3\text{ s}}$$

The distance the car travels during this time is $d = (95\text{ km/h})(6.47\times10^{-3}\text{ h}) = \boxed{0.61\text{ km}}$.

23)

Slightly different answers may be obtained since the data comes from reading the graph.

(*a*) The greatest velocity is found at the highest point on the graph, which is at $\boxed{t \approx 48\text{ s}}$.

(*b*) The indication of a constant velocity on a velocity–time graph is a slope of 0, which occurs from $\boxed{t = 90\text{ s to } t \approx 108\text{ s}}$.

(*c*) The indication of a constant acceleration on a velocity–time graph is a constant slope, which occurs from $\boxed{t = 0\text{ s to } t \approx 42\text{ s}}$, again from $\boxed{t \approx 65\text{ s to } t \approx 83\text{ s}}$, and again from $\boxed{t = 90\text{ s to } t \approx 108\text{ s}}$.

(*d*) The magnitude of the acceleration is greatest when the magnitude of the slope is greatest, which occurs from $\boxed{t \approx 65\text{ s to } t \approx 83\text{ s}}$.

29)

(*a*) Since the units of A times the units of t must equal meters, the units of A must be $\boxed{\text{m/s}}$.

Since the units of B times the units of t^2 must equal meters, the units of B must be $\boxed{\text{m/s}^2}$.

(*b*) The acceleration is the second derivative of the position function.

$$x = At + Bt^2 \quad \rightarrow \quad v = \frac{dx}{dt} = A + 2Bt \quad \rightarrow \quad a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = \boxed{2B\,\mathrm{m/s^2}}$$

(*c*) $v = A + 2Bt \quad \rightarrow \quad v(5) = \boxed{(A + 10B)\,\mathrm{m/s}} \qquad a = \boxed{2B\,\mathrm{m/s^2}}$

(*d*) The velocity is the derivative of the position function.

$$x = At + Bt^{-3} \quad \rightarrow \quad v = \frac{dx}{dt} = \boxed{A - 3Bt^{-4}}$$

37)

The words "slows down uniformly" implies that the car has a constant acceleration. The distance of travel is found from combining Eqs. 2-2 and 2-9.

$$x - x_0 = \frac{v_0 + v}{2}t = \left(\frac{18.0\,\mathrm{m/s} + 0\,\mathrm{m/s}}{2}\right)(5.00\ \mathrm{sec}) = \boxed{45.0\,\mathrm{m}}$$

51)

Choose upward to be the positive direction, and take $y_0 = 0$ to be at the height where the ball was hit. For the upward path, $v_0 = 20\,\mathrm{m/s}$, $v = 0$ at the top of the path, and $a = -9.80\,\mathrm{m/s^2}$.

(*a*) The displacement can be found from Eq. 2-12c, with x replaced by y.

$$v^2 = v_0^2 + 2a(y - y_0) \quad \rightarrow \quad y = y_0 + \frac{v^2 - v_0^2}{2a} = 0 + \frac{0 - (20\,\mathrm{m/s})^2}{2(-9.80\,\mathrm{m/s^2})} = \boxed{20\,\mathrm{m}}$$

(*b*) The time of flight can be found from Eq. 2-12b, with x replaced by y, using a displacement of 0 for the displacement of the ball returning to the height from which it was hit.

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 = 0 \quad \rightarrow \quad t(v_0 + \tfrac{1}{2}at) = 0 \quad \rightarrow \quad t = 0,\ t = \frac{2v_0}{-a} = \frac{2(20\,\mathrm{m/s})}{9.80\,\mathrm{m/s^2}} = \boxed{4\,\mathrm{s}}$$

The result of $t = 0$ s is the time for the original displacement of zero (when the ball was hit), and the result of $t = 4$ s is the time to return to the original displacement. Thus the answer is $t = 4$ s.

58)

(*a*) Choose upward to be the positive direction, and $y_0 = 0$ at the ground. The rocket has $v_0 = 0$, $a = 3.2\,\mathrm{m/s^2}$, and $y = 950\,\mathrm{m}$ when it runs out of fuel. Find the velocity of the rocket when it runs out of fuel from Eq 2-12c, with x replaced by y.

$$v_{950\,\mathrm{m}}^2 = v_0^2 + 2a(y - y_0) \quad \rightarrow$$

$$v_{950\,\mathrm{m}} = \pm\sqrt{v_0^2 + 2a(y - y_0)} = \pm\sqrt{0 + 2(3.2\,\mathrm{m/s^2})(950\ \mathrm{m})} = 77.97\,\mathrm{m/s} \approx \boxed{78\,\mathrm{m/s}}$$

The positive root is chosen since the rocket is moving upwards when it runs out of fuel.

(*b*) The time to reach the 950 m location can be found from Eq. 2-12a.

$$v_{950\,\mathrm{m}} = v_0 + at_{950\,\mathrm{m}} \quad \rightarrow \quad t_{950\,\mathrm{m}} = \frac{v_{950\,\mathrm{m}} - v_0}{a} = \frac{77.97\,\mathrm{m/s} - 0}{3.2\,\mathrm{m/s^2}} = 24.37\ \mathrm{s} \approx \boxed{24\ \mathrm{s}}$$

(*c*) For this part of the problem, the rocket will have an initial velocity $v_0 = 77.97\,\mathrm{m/s}$, an acceleration of $a = -9.80\,\mathrm{m/s^2}$, and a final velocity of $v = 0$ at its maximum altitude. The altitude reached from the out-of-fuel point can be found from Eq. 2-12c.

$$v^2 = v_{950\,\mathrm{m}}^2 + 2a(y - 950\ \mathrm{m}) \quad \rightarrow$$

$$y_{\max} = 950\ \mathrm{m} + \frac{0 - v_{950\,\mathrm{m}}^2}{2a} = 950\ \mathrm{m} + \frac{-(77.97\,\mathrm{m/s})^2}{2(-9.80\,\mathrm{m/s^2})} = 950\ \mathrm{m} + 310\ \mathrm{m} = \boxed{1260\ \mathrm{m}}$$

(*d*) The time for the "coasting" portion of the flight can be found from Eq. 2-12a.

$$v = v_{950\text{ m}} + at_{\text{coast}} \rightarrow t_{\text{coast}} = \frac{v - v_0}{a} = \frac{0 - 77.97\text{ m/s}}{-9.80\text{ m/s}^2} = 7.96\text{ s}$$

Thus the total time to reach the maximum altitude is $t = 24.37\text{ s} + 7.96\text{ s} = 32.33\text{ s} \approx \boxed{32\text{ s}}$.

(*e*) For the falling motion of the rocket, $v_0 = 0\text{ m/s}$, $a = -9.80\text{ m/s}^2$, and the displacement is -1260 m (it falls from a height of 1260 m to the ground). Find the velocity upon reaching the Earth from Eq. 2-12c.

$$v^2 = v_0^2 + 2a(y - y_0) \rightarrow$$

$$v = \pm\sqrt{v_0^2 + 2a(y - y_0)} = \pm\sqrt{0 + 2(-9.80\text{ m/s}^2)(-1260\text{ m})} = -157\text{ m/s} \approx \boxed{-160\text{ m/s}}$$

The negative root was chosen because the rocket is moving downward, which is the negative direction.

(*f*) The time for the rocket to fall back to the Earth is found from Eq. 2-12a.

$$v = v_0 + at \rightarrow t_{\text{fall}} = \frac{v - v_0}{a} = \frac{-157\text{ m/s} - 0}{-9.80\text{ m/s}^2} = 16.0\text{ s}$$

Thus the total time for the entire flight is $t = 32.33\text{ s} + 16.0\text{ s} = 48.33\text{ s} \approx \boxed{48\text{ s}}$. .

66)

(*a*) Choose up to be the positive direction. Let the throwing height of both objects be the $y = 0$ location, and so $y_0 = 0$ for both objects. The acceleration of both objects is $a = -g$. The equation of motion for the rock, using Eq. 2-12b, is $y_{\text{rock}} = y_0 + v_{0\text{ rock}}t + \frac{1}{2}at^2 = v_{0\text{ rock}}t - \frac{1}{2}gt^2$, where *t* is the time elapsed from the throwing of the rock. The equation of motion for the ball, being thrown 1.00 s later, is $y_{\text{ball}} = y_0 + v_{0\text{ ball}}(t - 1.00\text{ s}) + \frac{1}{2}a(t - 1.00\text{ s})^2 =$ $v_{0\text{ ball}}(t - 1.00\text{ s}) - \frac{1}{2}g(t - 1.00\text{ s})^2$. Set the two equations equal (meaning the two objects are at the same place) and solve for the time of the collision.

$$y_{\text{rock}} = y_{\text{ball}} \rightarrow v_{0\text{ rock}}t - \frac{1}{2}gt^2 = v_{0\text{ ball}}(t - 1.00\text{ s}) - \frac{1}{2}g(t - 1.00\text{ s})^2 \rightarrow$$

$$(12.0\text{ m/s})t - \frac{1}{2}(9.80\text{ m/s}^2)t^2 = (18.0\text{ m/s})(t - 1.00\text{ s}) - \frac{1}{2}(9.80\text{ m/s}^2)(t - 1.00\text{ s})^2 \rightarrow$$

$$(15.8\text{ m/s})t = (22.9\text{ m}) \rightarrow t = \boxed{1.45\text{ s}}$$

(*b*) Use the time for the collision to find the position of either object.

$$y_{\text{rock}} = v_{0\text{ rock}}t - \frac{1}{2}gt^2 = (12.0\text{ m/s})(1.45\text{ s}) - \frac{1}{2}(9.80\text{ m/s}^2)(1.45\text{ s})^2 = \boxed{7.10\text{ m}}$$

(*c*) Now the ball is thrown first, and so $y_{\text{ball}} = v_{0\text{ ball}}t - \frac{1}{2}gt^2$ and $y_{\text{rock}} = v_{0\text{ rock}}(t - 1.00\text{ s}) - \frac{1}{2}g(t - 1.00\text{ s})^2$. Again set the two equations equal to find the time of collision.

$$y_{\text{ball}} = y_{\text{rock}} \rightarrow v_{0\text{ ball}}t - \frac{1}{2}gt^2 = v_{0\text{ rock}}(t - 1.00\text{ s}) - \frac{1}{2}g(t - 1.00\text{ s})^2 \rightarrow$$

$$(18.0\text{ m/s})t - \frac{1}{2}(9.80\text{ m/s}^2)t^2 = (12.0\text{ m/s})(t - 1.00\text{ s}) - \frac{1}{2}(9.80\text{ m/s}^2)(t - 1.00\text{ s})^2 \rightarrow$$

$$(3.80\text{ m/s})t = 16.9\text{ m} \rightarrow t = 4.45\text{ s}$$

But this answer can be deceptive. Where do the objects collide?

$$y_{\text{ball}} = v_{0\text{ ball}}t - \frac{1}{2}gt^2 = (18.0\text{ m/s})(4.45\text{ s}) - \frac{1}{2}(9.80\text{ m/s}^2)(4.45\text{ s})^2 = -16.9\text{ m}$$

Thus, assuming they were thrown from ground level, they collide below ground level, which cannot happen. Thus $\boxed{\text{they never collide}}$.

68)

(*a*) The speed is the integral of the acceleration.

$$a=\frac{dv}{dt} \rightarrow dv=a\,dt \rightarrow dv=A\sqrt{t}\,dt \rightarrow \int_{v_0}^{v} dv = A\int_0^t \sqrt{t}\,dt \rightarrow$$

$$v-v_0=\tfrac{2}{3}At^{3/2} \rightarrow v=v_0+\tfrac{2}{3}At^{3/2} \rightarrow \boxed{v=7.5\,\mathrm{m/s}+\tfrac{2}{3}\left(2.0\,\mathrm{m/s^{5/2}}\right)t^{3/2}}$$

(*b*) The displacement is the integral of the velocity.

$$v=\frac{dx}{dt} \rightarrow dx=v\,dt \rightarrow dx=\left(v_0+\tfrac{2}{3}At^{3/2}\right)dt \rightarrow$$

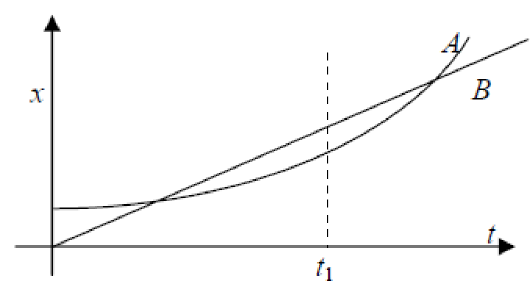
$$\int_{0\,\mathrm{m}}^{x} dx=\int_0^t\left(v_0+\tfrac{2}{3}At^{3/2}\right)dt \rightarrow x=v_0t+\tfrac{2}{3}\tfrac{2}{5}At^{5/2}=\boxed{\left(7.5\,\mathrm{m/s}\right)t+\tfrac{4}{15}\left(2.0\,\mathrm{m/s^{5/2}}\right)t^{5/2}}$$

(*c*) $a\left(t=5.0\,\mathrm{s}\right)=\left(2.0\,\mathrm{m/s^{5/2}}\right)\sqrt{5.0\,\mathrm{s}}=\boxed{4.5\,\mathrm{m/s^2}}$

$v\left(t=5.0\,\mathrm{s}\right)=7.5\,\mathrm{m/s}+\tfrac{2}{3}\left(2.0\,\mathrm{m/s^{5/2}}\right)\left(5.0\,\mathrm{s}\right)^{3/2}=22.41\,\mathrm{m/s}\approx\boxed{22\,\mathrm{m/s}}$

$x\left(t=5.0\,\mathrm{s}\right)=\left(7.5\,\mathrm{m/s}\right)\left(5.0\,\mathrm{s}\right)+\tfrac{4}{15}\left(2.0\,\mathrm{m/s^{5/2}}\right)\left(5.0\,\mathrm{s}\right)^{5/2}=67.31\,\mathrm{m}\approx\boxed{67\,\mathrm{m}}$

93)



(*a*) The two bicycles will have the same velocity at any time when the instantaneous slopes of their *x* vs. *t* graphs are the same. That occurs near the time t_1 as marked on the graph.

(*b*) Bicycle A has the larger acceleration, because its graph is concave upward, indicating a positive acceleration. Bicycle B has no acceleration because its graph has a constant slope.

(*c*) The bicycles are passing each other at the times when the two graphs cross, because they both have the same position at that time. The graph with the steepest slope is the faster bicycle, and so is the one that is passing at that instant. So at the first crossing, bicycle B is passing bicycle A. At the second crossing, bicycle A is passing bicycle B.

(*d*) Bicycle B has the highest instantaneous velocity at all times until the time t_1, where both graphs have the same slope. For all times after t_1, bicycle A has the highest instantaneous velocity. The largest instantaneous velocity is for bicycle A at the latest time shown on the graph.

(*e*) The bicycles appear to have the same average velocity. If the starting point of the graph for a particular bicycle is connected to the ending point with a straight line, the slope of that line is the average velocity. Both appear to have the same slope for that "average" line.