PHYS 4A – Mechanics by Prof. Hans Paar – Winter 2016

Chapter 11 Solutions

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1)

The angular momentum is given by Eq. 11-1.

$$L = I\omega = MR^2\omega = (0.210\,\text{kg})(1.35\,\text{m})^2(10.4\,\text{rad/s}) = \boxed{3.98\,\text{kg}\bullet\text{m}^2/\text{s}}$$

2)

(*a*) The angular momentum is given by Eq. 11-1.

$$L = I\omega = \tfrac{1}{2}MR^2\omega = \tfrac{1}{2}(2.8\,\text{kg})(0.18\,\text{m})^2\left[\left(\frac{1300\text{ rev}}{1\text{ min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\right]$$

$$= 6.175\,\text{kg}\bullet\text{m}^2/\text{s} \approx \boxed{6.2\,\text{kg}\bullet\text{m}^2/\text{s}}$$

(*b*) The torque required is the change in angular momentum per unit time. The final angular momentum is zero.

$$\tau = \frac{L - L_0}{\Delta t} = \frac{0 - 6.175\,\text{kg}\bullet\text{m}^2/\text{s}}{6.0\text{ s}} = \boxed{-1.0\,\text{m}\bullet\text{N}}$$

The negative sign indicates that the torque is used to oppose the initial angular momentum.

3)

(*a*) Consider the person and platform a system for angular momentum analysis. Since the force and torque to raise and/or lower the arms is internal to the system, the raising or lowering of the arms will cause no change in the total angular momentum of the system. However, the rotational inertia increases when the arms are raised. Since angular momentum is conserved, an increase in rotational inertia must be accompanied by a decrease in angular velocity.

(*b*) $L_i = L_f \;\rightarrow\; I_i\omega_i = I_f\omega_f \;\rightarrow\; I_f = I_i\dfrac{\omega_i}{\omega_f} = I_i\dfrac{0.90\,\text{rev/s}}{0.70\,\text{rev/s}} = 1.286 I_i \approx 1.3 I_i$

The rotational inertia has increased by a factor of $\boxed{1.3}$.

15)

Since there are no external torques on the system, the angular momentum of the 2-disk system is conserved. The two disks have the same final angular velocity.

$$L_i = L_f \;\rightarrow\; I\omega + I(0) = 2I\omega_f \;\rightarrow\; \boxed{\omega_f = \tfrac{1}{2}\omega}$$

23)

Use the definitions of cross product and dot product, in terms of the angle between the two vectors.

$$\left|\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right| = \vec{\mathbf{A}}\cdot\vec{\mathbf{B}} \quad\rightarrow\quad AB\left|\sin\theta\right| = AB\cos\theta \quad\rightarrow\quad \left|\sin\theta\right| = \cos\theta$$

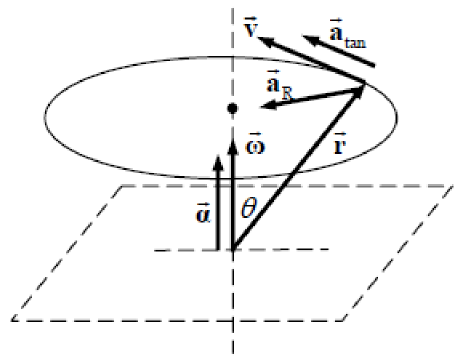
This is true only for angles with positive cosines, and so the angle must be in the first or fourth quadrant. Thus the solutions are $\theta = 45°, 315°$. But the angle between two vectors is always taken to be the smallest angle possible, and so $\theta = \boxed{45°}$.

25)

We choose coordinates so that the plane in which the particle rotates is the x-y plane, and so the angular velocity is in the z direction. The object is rotating in a circle of radius $r\sin\theta$, where θ is the angle between the position vector and the axis of rotation. Since the object is rigid and rotates about a fixed axis, the linear and angular velocities of the particle are related by $v = \omega r\sin\theta$. The magnitude of the tangential acceleration is $a_{\tan} = \alpha r\sin\theta$. The radial acceleration is given by

$$a_R = \frac{v^2}{r\sin\theta} = v\frac{v}{r\sin\theta} = v\omega.$$

We assume the object is gaining speed. See the diagram showing the various vectors involved.



The velocity and tangential acceleration are parallel to each other, and the angular velocity and angular acceleration are parallel to each other. The radial acceleration is perpendicular to the velocity, and the velocity is perpendicular to the angular velocity.

We see from the diagram that, using the right hand rule, the direction of $\vec{\mathbf{a}}_R$ is in the direction of $\vec{\boldsymbol{\omega}}\times\vec{\mathbf{v}}$. Also, since $\vec{\boldsymbol{\omega}}$ and $\vec{\mathbf{v}}$ are perpendicular, we have $\left|\vec{\boldsymbol{\omega}}\times\vec{\mathbf{v}}\right| = \omega v$ which from above is $v\omega = a_R$. Since both the magnitude and direction check out, we have $\boxed{\vec{\mathbf{a}}_R = \vec{\boldsymbol{\omega}}\times\vec{\mathbf{v}}}$.

We also see from the diagram that, using the right hand rule, the direction of $\vec{\mathbf{a}}_{\tan}$ is in the direction of $\vec{\boldsymbol{\alpha}}\times\vec{\mathbf{r}}$. The magnitude of $\vec{\boldsymbol{\alpha}}\times\vec{\mathbf{r}}$ is $\left|\vec{\boldsymbol{\alpha}}\times\vec{\mathbf{r}}\right| = \alpha r\sin\theta$, which from above is $\alpha r\sin\theta = a_{\tan}$. Since both the magnitude and direction check out, we have $\boxed{\vec{\mathbf{a}}_{\tan} = \vec{\boldsymbol{\alpha}}\times\vec{\mathbf{r}}}$.

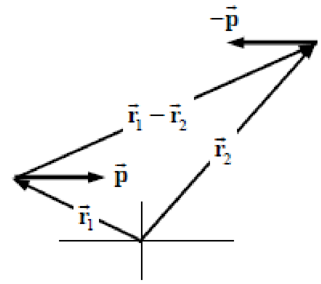
33)

The position vector and velocity vectors are at right angles to each other for circular motion. The angular momentum for a particle moving in a circle is $L = rp\sin\theta = rmv\sin 90° = mrv$. The moment of inertia is $I = mr^2$.

$$\frac{L^2}{2I} = \frac{(mrv)^2}{2mr^2} = \frac{m^2r^2v^2}{2mr^2} = \frac{mv^2}{2} = \tfrac{1}{2}mv^2 = K$$

This is analogous to $K = \dfrac{p^2}{2m}$ relating kinetic energy, linear momentum, and mass.

35)



See the diagram. Calculate the total angular momentum about the origin.

$$\vec{\mathbf{L}} = \vec{\mathbf{r}}_1 \times \vec{\mathbf{p}} + \vec{\mathbf{r}}_2 \times (-\vec{\mathbf{p}}) = (\vec{\mathbf{r}}_1 - \vec{\mathbf{r}}_2) \times \vec{\mathbf{p}}$$

The position dependence of the total angular momentum only depends on the difference in the two position vectors. That difference is the same no matter where the origin is chosen, because it is the relative distance between the two particles.

41)

(*a*) We assume the system is moving such that mass B is moving down, mass A is moving to the left, and the pulley is rotating counterclockwise. We take those as positive directions. The angular momentum of masses A and B is the same as that of a point mass. We assume the rope is moving without slipping, so $v = \omega_{\text{pulley}} R_0$.

$$L = L_A + L_B + L_{\text{pulley}} = M_A v R_0 + M_B v R_0 + I\omega = M_A v R_0 + M_B v R_0 + I\frac{v}{R_0}$$

$$= \boxed{\left((M_A + M_B) R_0 + \frac{I}{R_0} \right) v}$$

(*b*) The net torque about the axis of the pulley is that provided by gravity, $M_B g R_0$. Use Eq. 11-9, which is applicable since the axis is fixed.

$$\sum \tau = \frac{dL}{dt} \rightarrow M_B g R_0 = \frac{d}{dt}\left((M_A + M_B) R_0 + \frac{I}{R_0} \right) v = \left((M_A + M_B) R_0 + \frac{I}{R_0} \right) a \rightarrow$$

$$a = \frac{M_B g R_0}{\left((M_A + M_B) R_0 + \frac{I}{R_0} \right)} = \boxed{\frac{M_B g}{M_A + M_B + \frac{I}{R_0^2}}}$$

51)

Linear momentum of the center of mass is conserved in the totally inelastic collision.

$$p_{\text{initial}} = p_{\text{final}} \rightarrow mv = (m + M) v_{\substack{\text{CM}\\\text{final}}} \rightarrow \boxed{v_{\substack{\text{CM}\\\text{final}}} = \frac{mv}{m + M}}$$

Angular momentum about the center of mass of the system is conserved. First we find the center of mass, relative to the center of mass of the rod, taking up as the positive direction. See the diagram.

$$y_{\text{CM}} = \frac{m\left(\frac{1}{4}\ell\right) + M(0)}{(m + M)} = \frac{m\ell}{4(m + M)}$$

The distance of the stuck clay ball from the system's center of mass is found.

$$y_{\text{clay}} = \tfrac{1}{4}\ell - y_{\text{CM}} = \tfrac{1}{4}\ell - \frac{m\ell}{4(m + M)} = \frac{M\ell}{4(m + M)}$$

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We need the moment of inertia of the rod about the center of mass of the entire system. Use the parallel axis theorem. Treat the clay as a point mass.

$$I_{\text{rod}} = \tfrac{1}{12}M\ell^2 + M\left[\frac{m\ell}{4(m+M)}\right]^2$$

Now express the conservation of angular momentum about the system's center of mass.

$$L_{\text{initial}} = L_{\text{final}} \rightarrow mvy_{\text{clay}} = \left(I_{\text{rod}} + I_{\text{clay}}\right)\omega_{\text{final}} \rightarrow$$

$$\omega_{\text{final}} = \frac{mvy_{\text{clay}}}{\left(I_{\text{rod}} + I_{\text{clay}}\right)} = \frac{mvy_{\text{clay}}}{\left(\tfrac{1}{12}M\ell^2 + M\left[\frac{m\ell}{4(m+M)}\right]^2 + my_{\text{clay}}^2\right)}$$

$$= \frac{mv\dfrac{M\ell}{4(m+M)}}{\left(\tfrac{1}{12}M\ell^2 + M\left[\frac{m\ell}{4(m+M)}\right]^2 + m\left[\frac{M\ell}{4(m+M)}\right]^2\right)} = \frac{12mv(m+M)}{\ell\left(7m^2 + 11mM + 4M^2\right)}$$

$$= \boxed{\frac{12mv}{\ell(7m+4M)}}$$

54)

(*a*) The period of precession is related to the reciprocal of the angular precessional frequency.

$$T = \frac{2\pi}{\Omega} = \frac{2\pi I\omega}{Mgr} = \frac{2\pi\left[\tfrac{1}{2}Mr_{\text{disk}}^2\right]2\pi f}{Mgr} = \frac{2\pi^2 fr_{\text{disk}}^2}{gr} = \frac{2\pi^2(45\,\text{rev/s})(0.055\,\text{m})^2}{(9.80\,\text{m/s}^2)(0.105\,\text{m})}$$

$$= 2.611\,\text{s} \approx \boxed{2.6\,\text{s}}$$

(*b*) Use the relationship $T = \dfrac{2\pi^2 fr_{\text{disk}}^2}{gr}$ derived above to see the effect on the period.

$$\frac{T_{\text{new}}}{T_{\text{original}}} = \frac{\dfrac{2\pi^2 fr_{\text{disk new}}^2}{gr_{\text{new}}}}{\dfrac{2\pi^2 fr_{\text{disk}}^2}{gr}} = \frac{\dfrac{r_{\text{disk new}}^2}{r_{\text{new}}}}{\dfrac{r_{\text{disk}}^2}{r}} = \left(\frac{r_{\text{disk new}}}{r_{\text{disk}}}\right)^2 \frac{r}{r_{\text{new}}} = \left(\frac{2}{1}\right)^2 \frac{1}{2} = 2$$

So the period would double, and thus be $T_{\text{new}} = 2T_{\text{original}} = 2(2.611\,\text{s}) = 5.222\,\text{s} \approx \boxed{5.2\,\text{s}}$.

55)

Use Eq. 11-13c for the precessional angular velocity.

$$\Omega = \frac{Mgr}{I\omega} = \frac{Mg\left(\tfrac{1}{2}\ell_{\text{axle}}\right)}{\tfrac{1}{2}Mr_{\text{wheel}}^2\omega} = \frac{g\ell_{\text{axle}}}{r_{\text{wheel}}^2\omega} = \frac{(9.80\,\text{m/s}^2)(0.25\,\text{m})}{(0.060\,\text{m})^2(85\,\text{rad/s})} = \boxed{8.0\,\text{rad/s}} \quad (1.3\,\text{rev/s})$$

56)

The mass is placed on the axis of rotation and so does not change the moment of inertia. The addition of the mass does change the center of mass position r, and it does change the total mass, M, to $\frac{3}{2}M$.

$$r_{\text{new}} = \frac{M\left(\frac{1}{2}\ell_{\text{axle}}\right) + \frac{1}{2}M\ell_{\text{axle}}}{M + \frac{1}{2}M} = \frac{M\ell_{\text{axle}}}{\frac{3}{2}M} = \frac{2}{3}\ell_{\text{axle}}$$

$$\frac{\Omega_{\text{new}}}{\Omega_{\text{original}}} = \frac{\dfrac{M_{\text{new}} g r_{\text{new}}}{I\omega}}{\dfrac{M_{\text{original}} g r_{\text{original}}}{I\omega}} = \frac{\frac{3}{2}M\left(\frac{2}{3}\ell_{\text{axle}}\right)}{M\left(\frac{1}{2}\ell_{\text{axle}}\right)} = 2 \quad \rightarrow$$

$$\Omega_{\text{new}} = 2\Omega_{\text{original}} = 2(8.0\,\text{rad/s}) = \boxed{16\,\text{rad/s}}$$