PHYS 4A – Mechanics by Prof. Hans Paar – Winter 2016

Chapter 10 Solutions

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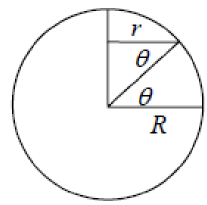
9)

Each location will have the same angular velocity (1 revolution per day), but the radius of the circular path varies with the location. From the diagram, we see $r = R\cos\theta$, where R is the radius of the Earth, and r is the radius at latitude θ.



$$(a)\quad v = \omega r = \frac{2\pi}{T}r = \left(\frac{2\pi\,\text{rad}}{1\,\text{day}}\right)\left(\frac{1\,\text{day}}{86400\,\text{s}}\right)\left(6.38\times10^{6}\,\text{m}\right) = \boxed{464\,\text{m/s}}$$

$$(b)\quad v = \omega r = \frac{2\pi}{T}r = \left(\frac{2\pi\,\text{rad}}{1\,\text{day}}\right)\left(\frac{1\,\text{day}}{86400\,\text{s}}\right)\left(6.38\times10^{6}\,\text{m}\right)\cos 66.5^\circ = \boxed{185\,\text{m/s}}$$

$$(c)\quad v = \omega r = \frac{2\pi}{T}r = \left(\frac{2\pi\,\text{rad}}{1\,\text{day}}\right)\left(\frac{1\,\text{day}}{86400\,\text{s}}\right)\left(6.38\times10^{6}\,\text{m}\right)\cos 45.0^\circ = \boxed{328\,\text{m/s}}$$

10)

(a) The Earth makes one orbit around the Sun in one year.

$$\omega_{\text{orbit}} = \frac{\Delta\theta}{\Delta t} = \left(\frac{2\pi\,\text{rad}}{1\,\text{year}}\right)\left(\frac{1\text{ year}}{3.16\times10^{7}\,\text{s}}\right) = \boxed{1.99\times10^{-7}\,\text{rad/s}}$$

(b) The Earth makes one revolution about its axis in one day.

$$\omega_{\text{rotation}} = \frac{\Delta\theta}{\Delta t} = \left(\frac{2\pi\,\text{rad}}{1\,\text{day}}\right)\left(\frac{1\,\text{day}}{86{,}400\,\text{s}}\right) = \boxed{7.27\times10^{-5}\,\text{rad/s}}$$

23)

(a) The angular velocity is found by integrating the angular acceleration function.

$$\alpha = \frac{d\omega}{dt} \;\rightarrow\; d\omega = \alpha dt \;\rightarrow\; \int_0^{\omega} d\omega = \int_0^t \alpha dt = \int_0^t \left(5.0t^2 - 8.5t\right)dt \;\rightarrow\; \boxed{\omega = \tfrac{1}{3}5.0t^3 - \tfrac{1}{2}8.5t^2}$$

(b) The angular position is found by integrating the angular velocity function.

$$\omega = \frac{d\theta}{dt} \;\rightarrow\; d\theta = \omega dt \;\rightarrow\; \int_0^{\theta} d\theta = \int_0^t \omega dt = \int_0^t \left(\tfrac{1}{3}5.0t^3 - \tfrac{1}{2}8.5t^2\right)dt \;\rightarrow$$

$$\boxed{\theta = \tfrac{1}{12}5.0t^4 - \tfrac{1}{6}8.5t^3}$$

$$(c)\quad \omega(2.0\,\text{s}) = \tfrac{1}{3}5.0(2.0)^3 - \tfrac{1}{2}8.5(2.0)^2 = -3.7\,\text{rad/s} \approx \boxed{-4\,\text{rad/s}}$$

$$\theta(2.0\,\text{s}) = \tfrac{1}{12}5.0(2.0)^4 - \tfrac{1}{6}8.5(2.0)^3 = -4.67\,\text{rad} \approx \boxed{-5\,\text{rad}}$$

37)

(*a*) The small ball can be treated as a particle for calculating its moment of inertia.

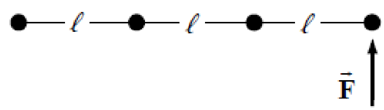
$$I = MR^2 = (0.650\,\text{kg})(1.2\,\text{m})^2 = \boxed{0.94\,\text{kg}\cdot\text{m}^2}$$

(*b*) To keep a constant angular velocity, the net torque must be zero, and so the torque needed is the same magnitude as the torque caused by friction.

$$\sum \tau = \tau_{\text{applied}} - \tau_{\text{fr}} = 0 \quad \rightarrow \quad \tau_{\text{applied}} = \tau_{\text{fr}} = F_{\text{fr}} r = (0.020\,\text{N})(1.2\,\text{m}) = \boxed{2.4 \times 10^{-2}\,\text{m}\cdot\text{N}}$$

45)

Each mass is treated as a point particle. The first mass is at the axis of rotation; the second mass is a distance ℓ from the axis of rotation; the third mass is 2ℓ from the axis, and the fourth mass is 3ℓ from the axis.



(*a*) $I = M\ell^2 + M(2\ell)^2 + M(3\ell)^2 = \boxed{14M\ell^2}$

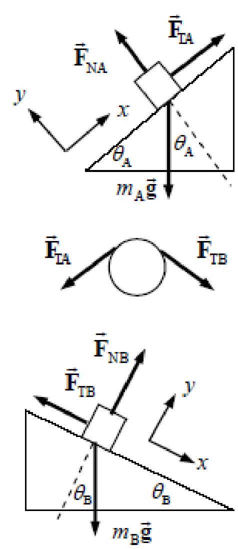
(*b*) The torque to rotate the rod is the perpendicular component of force times the lever arm, and is also the moment of inertia times the angular acceleration.

$$\tau = I\alpha = F_\perp r \quad \rightarrow \quad F_\perp = \frac{I\alpha}{r} = \frac{14M\ell^2\alpha}{3\ell} = \boxed{\tfrac{14}{3}M\ell\alpha}$$

(*c*) The force must be perpendicular to the rod connecting the masses, and perpendicular to the axis of rotation. An appropriate direction is shown in the diagram.

46)

(*a*) The free body diagrams are shown. Note that only the forces producing torque are shown on the pulley. There would also be a gravity force on the pulley (since it has mass) and a normal force from the pulley's suspension, but they are not shown.



(*b*) Write Newton's second law for the two blocks, taking the positive x direction as shown in the free body diagrams.

$$m_A: \quad \sum F_x = F_{TA} - m_A g \sin\theta_A = m_A a \quad \rightarrow$$

$$F_{TA} = m_A(g\sin\theta_A + a)$$

$$= (8.0\text{kg})\left[(9.80\,\text{m/s}^2)\sin 32^\circ + 1.00\,\text{m/s}^2\right] = 49.55\,\text{N}$$

$$\approx \boxed{50\,\text{N}} \quad (2 \text{ sig fig})$$

$$m_B: \quad \sum F_x = m_B g \sin\theta_B - F_{TB} = m_B a \quad \rightarrow$$

$$F_{TB} = m_B(g\sin\theta_B - a)$$

$$= (10.0\text{kg})\left[(9.80\,\text{m/s}^2)\sin 61^\circ - 1.00\,\text{m/s}^2\right] = 75.71\,\text{N}$$

$$\approx \boxed{76\,\text{N}}$$

(*c*) The net torque on the pulley is caused by the two tensions. We take clockwise torques as positive.

$$\sum \tau = (F_{TB} - F_{TB})R = (75.71\text{N} - 49.55\text{N})(0.15\text{m}) = 3.924\text{m}\cdot\text{N} \approx \boxed{3.9\text{m}\cdot\text{N}}$$

Use Newton's second law to find the rotational inertia of the pulley. The tangential acceleration of the pulley's rim is the same as the linear acceleration of the blocks, assuming that the string doesn't slip.

$$\sum \tau = I\alpha = I\frac{a}{R} = (F_{TB} - F_{TB})R \quad \rightarrow$$

$$I = \frac{(F_{TB} - F_{TB})R^2}{a} = \frac{(75.71\text{N} - 49.55\text{N})(0.15\text{m})^2}{1.00\,\text{m/s}^2} = \boxed{0.59\,\text{kg}\cdot\text{m}^2}$$

64)

To maintain a constant angular speed ω_{steady} will require a torque τ_{motor} to oppose the frictional torque. The power required by the motor is $P = \tau_{\text{motor}}\omega_{\text{steady}} = -\tau_{\text{friction}}\omega_{\text{steady}}$.

$$\tau_{\text{friction}} = I\alpha_{\text{friction}} = \tfrac{1}{2}MR^2\left(\frac{\omega_f - \omega_0}{t}\right) \rightarrow$$

$$P_{\text{motor}} = \tfrac{1}{2}MR^2\left(\frac{\omega_0 - \omega_f}{t}\right)\omega_{\text{steady}} = \tfrac{1}{2}(220\,\text{kg})(5.5\,\text{m})^2\frac{\left[(3.8\,\text{rev/s})\left(\frac{2\pi\,\text{rad}}{\text{rev}}\right)\right]^2}{16\,\text{s}} = 1.186\times10^5\,\text{W}$$

$$= 1.186\times10^5\,\text{W}\left(\frac{1\,\text{hp}}{746\,\text{W}}\right) = 158.9\,\text{hp} \approx \boxed{160\,\text{hp}}$$

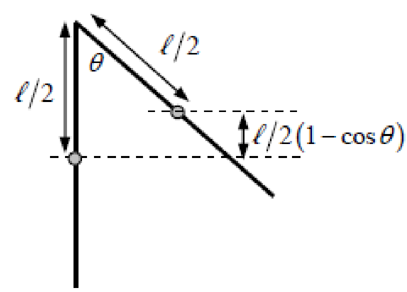
65)

The work required is the change in rotational kinetic energy. The initial angular velocity is 0.

$$W = \Delta K_{\text{rot}} = \tfrac{1}{2}I\omega_f^2 - \tfrac{1}{2}I\omega_i^2 = \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\omega_f^2 = \tfrac{1}{4}(1640\,\text{kg})(7.50\,\text{m})^2\left(\frac{2\pi\,\text{rad}}{8.00\,\text{s}}\right)^2 = \boxed{1.42\times10^4\,\text{J}}$$

66)

Mechanical energy will be conserved. The rotation is about a fixed axis, so $K_{\text{tot}} = K_{\text{rot}} = \tfrac{1}{2}I\omega^2$. For gravitational potential energy, we can treat the object as if all of its mass were at its center of mass. Take the lowest point of the center of mass as the zero location for gravitational potential energy.



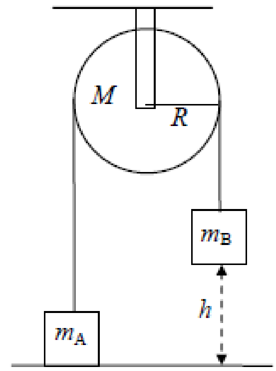
$$E_{\text{initial}} = E_{\text{final}} \rightarrow U_{\text{initial}} = K_{\text{final}} \rightarrow$$

$$Mg\tfrac{1}{2}\ell(1-\cos\theta) = \tfrac{1}{2}I\omega_{\text{bottom}}^2 = \tfrac{1}{2}\left(\tfrac{1}{3}M\ell^2\right)\omega_{\text{bottom}}^2 \rightarrow$$

$$\omega_{\text{bottom}} = \boxed{\sqrt{\frac{3g}{\ell}(1-\cos\theta)}} \; ; \; v_{\text{bottom}} = \omega_{\text{bottom}}\ell = \boxed{\sqrt{3g\ell(1-\cos\theta)}}$$

67)

The only force doing work in this system is gravity, so mechanical energy is conserved. The initial state of the system is the configuration with m_A on the ground and all objects at rest. The final state of the system has m_B just reaching the ground, and all objects in motion. Call the zero level of gravitational potential energy to be the ground level. Both masses will have the same speed since they are connected by the rope. Assuming that the rope does not slip on the pulley, the angular speed of the pulley is related to the speed of the masses by $\omega = v/R$. All objects have an initial speed of 0.



$$E_i = E_f \quad \rightarrow$$

$$\tfrac{1}{2}m_A v_i^2 + \tfrac{1}{2}m_B v_i^2 + \tfrac{1}{2}I\omega_i^2 + m_A g y_{1i} + m_B g y_{2i} = \tfrac{1}{2}m_A v_f^2 + \tfrac{1}{2}m_B v_f^2 + \tfrac{1}{2}I\omega_f^2 + m_A g y_{1f} + m_B g y_{2f}$$

$$m_B gh = \tfrac{1}{2}m_A v_f^2 + \tfrac{1}{2}m_B v_f^2 + \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\left(\frac{v_f^2}{R^2}\right) + m_A gh$$

$$v_f = \sqrt{\frac{2(m_B - m_A)gh}{(m_A + m_B + \tfrac{1}{2}M)}} = \sqrt{\frac{2(38.0\,\text{kg} - 35.0\,\text{kg})(9.80\,\text{m/s}^2)(2.5\,\text{m})}{(38.0\,\text{kg} + 35.0\,\text{kg} + (\tfrac{1}{2})3.1\,\text{kg})}} = \boxed{1.4\,\text{m/s}}$$

68)

(*a*) The kinetic energy of the system is the kinetic energy of the two masses, since the rod is treated as massless. Let A represent the heavier mass, and B the lighter mass.

$$K = \tfrac{1}{2}I_A\omega_A^2 + \tfrac{1}{2}I_B\omega_B^2 = \tfrac{1}{2}m_A r_A^2\omega_A^2 + \tfrac{1}{2}m_B r_B^2\omega_A^2 = \tfrac{1}{2}r^2\omega^2(m_A + m_B)$$

$$= \tfrac{1}{2}(0.210\,\text{m})^2(5.60\,\text{rad/s})^2(7.00\,\text{kg}) = \boxed{4.84\,\text{J}}$$

(*b*) The net force on each object produces centripetal motion, and so can be expressed as $mr\omega^2$.

$$F_A = m_A r_A \omega_A^2 = (4.00\,\text{kg})(0.210\,\text{m})(5.60\,\text{rad/s})^2 = \boxed{26.3\,\text{N}}$$

$$F_B = m_B r_B \omega_B^2 = (3.00\,\text{kg})(0.210\,\text{m})(5.60\,\text{rad/s})^2 = \boxed{19.8\,\text{N}}$$

These forces are exerted by the rod. Since they are unequal, there would be a net horizontal force on the rod (and hence the axle) due to the masses. This horizontal force would have to be counteracted by the mounting for the rod and axle in order for the rod not to move horizontally. There is also a gravity force on each mass, balanced by a vertical force from the rod, so that there is no net vertical force on either mass.

(*c*) Take the 4.00 kg mass to be the origin of coordinates for determining the center of mass.

$$x_{CM} = \frac{m_A x_A + m_B x_B}{m_A + m_B} = \frac{(4.00\,\text{kg})(0) + (3.00\,\text{kg})(0.420\,\text{m})}{7.00\,\text{kg}} = 0.180\,\text{m from mass A}$$

So the distance from mass A to the axis of rotation is now 0.180 m, and the distance from mass B to the axis of rotation is now 0.24 m. Re-do the above calculations with these values.

$$K = \tfrac{1}{2}I_A\omega_A^2 + \tfrac{1}{2}I_B\omega_B^2 = \tfrac{1}{2}m_A r_A^2\omega_A^2 + \tfrac{1}{2}m_B r_B^2\omega_A^2 = \tfrac{1}{2}\omega^2(m_A r_A^2 + m_B r_B^2)$$

$$= \tfrac{1}{2}(5.60\,\text{rad/s})^2\left[(4.00\,\text{kg})(0.180\,\text{m})^2 + (3.00\,\text{kg})(0.240\,\text{m})^2\right] = \boxed{4.74\,\text{J}}$$

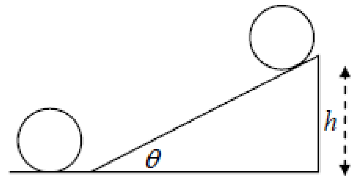
$$F_A = m_A r_A \omega_A^2 = (4.00\,\text{kg})(0.180\,\text{m})(5.60\,\text{rad/s})^2 = \boxed{22.6\,\text{N}}$$

$$F_B = m_B r_B \omega_B^2 = (3.00\,\text{kg})(0.240\,\text{m})(5.60\,\text{rad/s})^2 = \boxed{22.6\,\text{N}}$$

Note that the horizontal forces are now equal, and so there will be no horizontal force on the rod or axle.

92)

(*a*) Assuming that there are no dissipative forces doing work, conservation of mechanical energy may be used to find the final height h of the hoop. Take the bottom of the incline to be the zero level of gravitational potential energy. We assume that the hoop is rolling without sliding, so that $\omega = v/R$. Relate the conditions at the bottom of the incline to the conditions at the top by conservation of energy. The hoop has both translational and rotational kinetic energy at the bottom, and the rotational inertia of the hoop is given by $I = mR^2$.



$$E_{\text{bottom}} = E_{\text{top}} \rightarrow \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = mgh \rightarrow \tfrac{1}{2}mv^2 + \tfrac{1}{2}mR^2\frac{v^2}{R^2} = mgh \rightarrow$$

$$h = \frac{v^2}{g} = \frac{(3.3\,\text{m/s})^2}{9.80\,\text{m/s}^2} = 1.111\text{ m}$$

The distance along the plane is given by $d = \dfrac{h}{\sin\theta} = \dfrac{1.111\text{ m}}{\sin 15^\circ} = 4.293\text{ m} \approx \boxed{4.3\text{ m}}$

(*b*) The time can be found from the constant acceleration linear motion.

$$\Delta x = \tfrac{1}{2}(v + v_o)t \rightarrow t = \frac{2\Delta x}{v + v_o} = \frac{2(4.293\text{ m})}{0 + 3.3\,\text{m/s}} = 2.602\text{ s}$$

This is the time to go up the plane. The time to come back down the plane is the same, and so the total time is $\boxed{5.2\,\text{s}}$.