H.P. Paar **PHYS 4B: Mechanics, Fluids, Waves & Heat** Spring 2016

Statistical physics HW solutions
Solutions by Yury Kiselev

1. (a) Summing up all the heights and dividing by number of students, 10, gives 1.75.
   (b) Histogram is plotted in Fig. 1b. See the end of the file.
   (c) We will use middle values of height from the histogram to answer this question. The answer is $\langle h \rangle = \frac{1*1.55+2*1.65+3*1.75+3*1.85+1*1.95}{10} = 1.76\text{m}$.
   (d) From the histogram, it's 4 people out of 10, so 40%.
   (e) $1/10, 2/10, 3/10, 3/10, 1/10$ respectively.
   (f) 1.0.

2. (a) It's 32.
   (b) We will use middle values of velocities from the graph to answer this question. The answer is $\langle v \rangle = \frac{2*600+4*700+6*800+9*900+7*1000+3*1100+1*1300}{32} \approx 891$ mi/h
   (c) The answer is $\langle v^2 \rangle = \frac{2*600^2+4*700^2+6*800^2+9*900^2+7*1000^2+3*1100^2+1*1300^2}{32} \approx 817{\cdot}10^3\ \text{mi}^2/\text{h}^2$
   (d) It's defined as a square root of the latter, so

$$v_{rms} = \sqrt{\langle v^2 \rangle} \approx 904 \text{ mi/h}$$

   (e) No, it's not the same, $\langle v^2 \rangle \neq \langle v \rangle^2$.
   (f) We will use half of one bin and half of the other, so answer is $p = (4/2+6/2)/32 = 5/32 \approx 16\%$.
   (g) No atoms here, so zero.
   (h) $2/32, 4/32, 6/32, 9/32, 7/32, 3/32, 1/32$ respectively.

3. (a) The distance from the axis to the atoms is $l/2$, so $I = 2m(l/2)^2 = ml^2/2$.
   (b) The distance from the axis to the atoms is zero in this case, so $I = 0$.
   (c) $K_1 = \frac{1}{2}I_1\omega^2 = \frac{1}{2}I_1(2\pi)^2 f^2 = \pi^2 ml^2 f^2$.
   (d) $K_2 = 0$, because $I_2 = 0$.

4. Gases are in equilibrium, so they have same temperatures. $v_{rms} = \sqrt{\frac{3kT}{m}}$, so the ratio of the velocities is inversely proportional to the square root of the ratio of the mass of the molecules. Helium is a monoatomic gas, while hydrogen is diatomic, so

$$\frac{v_{He}}{v_{H2}} = \sqrt{\frac{m_{H2}}{m_{He}}} = \sqrt{\frac{2}{4}} = \frac{1}{\sqrt{2}}.$$

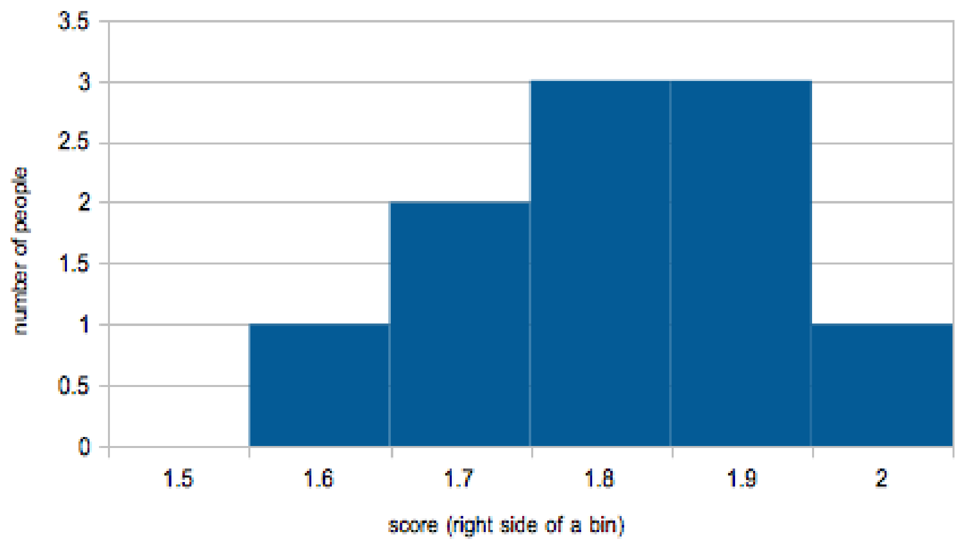
5. (a) $\langle v_z \rangle \propto \int_{-\infty}^{\infty} v_x \cdot e^{-mv_x^2/kT} \mathrm{d}v_x = 0$, because this integral is an odd function.

   (b) Boltzmann factors for other components will be different, $e^{-mv_x^2/kT}$ and $e^{-mv_y^2/kT}$ respectively, but the integral will look the same and will result in zero as well.

   (c) No, because $\langle v^2 \rangle = \langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle \neq \langle v_x \rangle^2 + \langle v_y \rangle^2 + \langle v_z \rangle^2$.

6. (a) Assuming that the vertical axis is z as usual, there is no energy associated with the difference in y coordinates, so the Boltzmann factor equals to 1. The average position is then $\langle y \rangle \propto \int_{y_{min}}^{y_{max}} 1 \cdot \mathrm{d}y = 0$.

   (b) This average is given by $\langle \vec{r} \rangle = (\langle x \rangle, \langle y \rangle, \langle z \rangle)$. I will assume that the container is small (not comparable to the size of the atmosphere), so the Boltzmann factors for all coordinates are equal to 1 (as $mgz \ll kT$). With this assumption, $\langle \vec{r} \rangle = (0, 0, 0)$ – center of the cube.


3.5
3
2.5
2
1.5
1
0.5
0
number of people
1.5
1.6
1.7
1.8
1.9
2
score (right side of a bin)