H.P. Paar **PHYS 4B: Mechanics, Fluids, Waves & Heat** Spring 2016

Quizz # 4 solutions
Solutions by Yury Kiselev

1. (20 points)

   (a) (4 points) Number of people in each bin is the height of each bin. Summing them up, we get that number of people $N = 250$.

   (b) (4 points) We need to sum up last 4 nonzero bins:

   $$p = \tfrac{55}{N} = \tfrac{55}{250} = 0.22.$$

   (c) (4 points) To calculate average height we may use $\langle h \rangle = (\sum_i h_i N_i)/N$, where h_i – the height, corresponding to the center of the bin. Instead, I will use left and right sides of each bin to calculate possible range of average height. Your result should fall into this range.
   Taking left side of each bin,

   $$\langle h \rangle_L = (\sum_i h_i N_i)/N = \tfrac{426.5}{250} = 1.706.$$

   If we take the right side of each bin, the value of each bin would change by 0.05, so the result also would change by 0.05. The resulting range is then

   $$\langle h \rangle = 1.73 \pm 0.03.$$

   (d) (4 points) We don't have any records for people above 2.2 m, so the probability is zero.

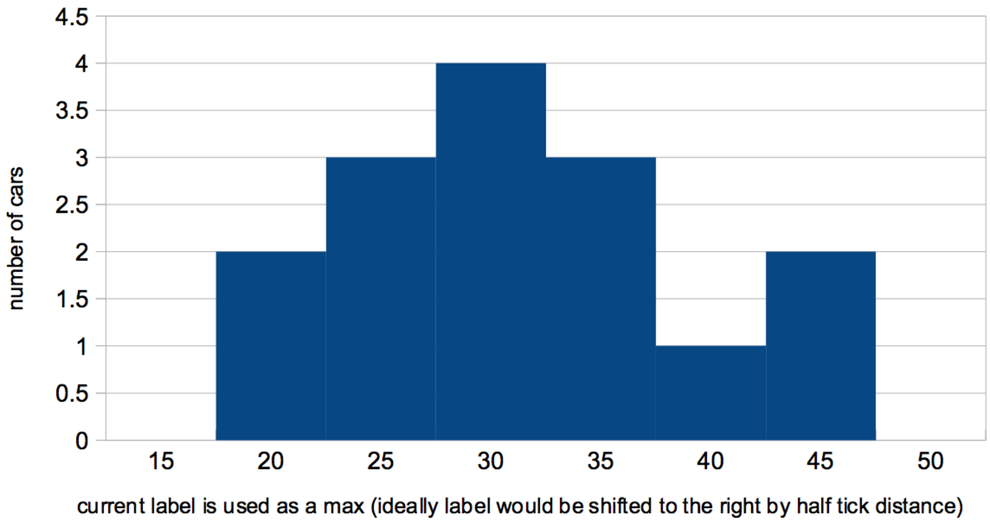
   (e) (4 points) We read directly from the histogram: $p = 30/N$, so

   $$p = \tfrac{30}{250} = 0.12.$$

2. (20 points)

   (a) (2 points) Average price $m = (\sum_{i=1}^{N} m_i)/N = 429/15 \cdot 10^3$ USD $= \$28,600$.

   (b) (3 points) Average price of US cars $m_{US} = (\sum_{i=1}^{N_{US}} m_i)/N_{US} = 102/5 \cdot 10^3$ USD $=$ $\$20,400$.



Average price of Germany cars $m_G = (\sum_{i=1}^{N_G} m_i)/N_G = 185/5 \cdot 10^3$ USD $= \$37,000$.

Average price of Japan cars $m_J = (\sum_{i=1}^{N_J} m_i)/N_J = 142/5 \cdot 10^3$ USD $= \$28,400$.

(c) (3 points) Histogram is plotted in Fig. 2c.

(d) (3 points) Let's use the middle value of the cost of each step of the histogram to calculate total mean value:

$$\langle m \rangle = (\sum_i m_i h_i)/N = \tfrac{432.5}{15} \approx \$28,833.$$

(e) (3 points) There are two ways to calculate this probability: from raw data or from the histogram. Looking at the raw data we see four cars below 22K, so the probability is $p = 4/15 \approx 0.27$.

If we use the histogram, we need to estimate number of cars representing part of the step: from 20K to 22K. We will assume that the distribution is even inside the step, so number of cars in the range 20K to 22K will be $N_{frac} = 2/5 \cdot 3$. The total number of cars below 22K is $N_{L22} = 2 + 2/5 \cdot 3$ and the probability is $p = N_{L22}/N = 3.2/15 \approx 0.21$. Quite different result, but also acceptable.

(f) (3 points) We need to use raw data here: We have 5 cars from Germany, only 1 of them costs less that 22K. We estimate the probability to be $p = 20\% = 0.2$, but we must understand that strictly speaking we need more statistical data to make this estimation (For example, 100 German cars).

(g) (3 points) With the same note in mind as in the previous point, we very roughly can estimate it as $p = 2/5$, because 2 cars out of 5 given fall into the given range. So, $p = 0.4 = 40\%$.

3. (20 points)

   (a) (5 points) Note about the constants: $N_A = 6 \cdot 10^{23}$ mol^{-1} $= 6 \cdot 10^{26}$ kmol^{-1} and $R = 8.3$ J/(mol $\cdot$ K) $= 8.3$ kJ/(kmol $\cdot$ K), so we can use any pair, depending on what units we use for amount of substance, moles or kilomoles.
   $PV = (N/N_A)RT$, so

$$P = \frac{NRT}{N_A V} = \frac{3 \cdot 10^{27} \cdot 8.3 \cdot 10^3 \cdot 27}{6 \cdot 10^{26} \cdot 2} = 5.6 \cdot 10^5 \text{ Pa.}$$

   (b) (5 points) Hydrogen gas is formed by molecules H_2, so the gas is diatomic. Average energy per molecule is proportional to kT, where constant of proportionality depends on temperature – most commonly it's 5/2, but in general can be 3/2, 5/2 or 7/2 (7 d.o.f = 3 center of mass motion + 2 rotations + 1 kinetic vibrational (v) + 1 potential vibrational (x) d.o.f.) [For more info see, for example, wikipedia article "Heat Capacity" or any book on the topic]. So, all answers below are acceptable

$$\langle E_1 \rangle = \tfrac{3}{2}kT = 1.5kT = 5.6 \cdot 10^{-22} \text{ J},$$
$$\langle E_1 \rangle = \tfrac{5}{2}kT = 9.3 \cdot 10^{-22} \text{ J},$$
$$\langle E_1 \rangle = \tfrac{7}{2}kT = 13.1 \cdot 10^{-22} \text{ J}.$$

   (c) (5 points) They are corresponding to three different total energies

$$E = N\langle E_1 \rangle = N\tfrac{3}{2}kT = 1.5NkT = 1.7 \cdot 10^6 \text{ J}$$
$$E = N\langle E_1 \rangle = N\tfrac{5}{2}kT = 2.8 \cdot 10^6 \text{ J},$$
$$E = N\langle E_1 \rangle = N\tfrac{7}{2}kT = 3.9 \cdot 10^6 \text{ J}.$$

   (d) (5 points) Using same formula as in part (a), we get

$$P = \frac{NRT}{N_A V} = \frac{3 \cdot 10^{27} \cdot 8.3 \cdot 10^3 \cdot 127}{6 \cdot 10^{26} \cdot 2} \approx 26 \cdot 10^5 \text{ Pa.}$$

4. (40 points)

   (a) (25 points) There are two ways to solve this problem. First is to notice that the pressure is proportional to number density: $P = NkT/V = nkT$, so pressure is proportional to number of molecules in a given volume, which is number density. Then, $P = const \cdot n_0 \cdot e^{-mgh/(kT)}$ and $const \cdot n_0 = P_0$, because at zero height the pressure is equal to P_0. So, $P = P_0 e^{-mgh/(kT)}$.

   The second way to solve it, is to calculate pressure due to the air column weight above certain height. Considering some finite horizontal area A at height h, the total force acting on it equals to the weight of the air above. $M_{above}(h) = \int_h^\infty A\rho dz = \int_h^\infty An(z)mdz = \int_h^\infty An_0 e^{-mgz/(kT)} mdz = An_0 m(kT)/(mg)e^{-mgh/(kT)} \propto Ae^{-mgh/(kT)}$. The pressure is them $P(h) \propto M_{above}(h)g/A \propto e^{-mgh/(kT)}$ and the constant in front of the exponent is again equal to P_0: $P = P_0 e^{-mgh/(kT)}$.

(b) (15 points) $P = P_0 e^{-mgh/(kT)} = P_0/2$, so $\ln(P_0) - mgh/(kT) = \ln(P_0/2) = \ln(P_0) - \ln(2)$. Then,

$$h = \tfrac{kT}{mg}\ln(2) \approx 0.69\tfrac{kT}{mg}.$$
