H.P. Paar **PHYS 4B: Mechanics, Fluids, Waves & Heat** Spring 2016

Quizz # 3 solutions

Solutions by Yury Kiselev

1. (15 points)

   (a) (7 points) We are given arrival clock times t_T and t_L. The only relevant information here is $\Delta t = t_T - t_L$. Let's denote travel times τ_L and τ_T respectively, so $L = \tau_L v_L$ and $L = \tau_T v_T = (\tau_L + \Delta t)v_T$. Solving these two equations with respect to τ_L and L gives us

   $$L = \frac{\Delta t \ v_T v_L}{v_L - v_T}.$$

   (b) (8 points) Distance between the station and epicenter defines a sphere with radius L: the epicenter can be located at any point of the sphere that lies on or below the ground. If we want to locate the exact epicenter, we will need additional stations. If we have one extra station, then we will be able to measure two distances that will define two spheres. Our epicenter should be located on the intersection between two spheres. This intersection is a circle in general and so we still can't localize the exact position of the epicenter. To do so we need a fourth station.

2. (15 points)

   (a) (5 points) Main source of sound is always fundamental frequency, which corresponds to $\lambda_1 = 2l = \lambda_2$.

   (b) (5 points) Velocity of the wave propagation is $v = \sqrt{T/\mu}$ and $\lambda = vT = v/f$, so

   $$f_1 = \tfrac{1}{2l}\sqrt{\tfrac{T_1}{\mu}}, \qquad f_2 = \tfrac{1}{2l}\sqrt{\tfrac{T_2}{\mu}}.$$

   (c) (5 points) Let's consider two sinusoidal waves that interfere which each other at fixed $x = 0$. $D_1 = A\sin 2\pi f_1 t$ and $D_2 = A \sin 2\pi f_2 t$. The resultant displacement can be written as

   $$D = D_1 + D_2 = A(\sin 2\pi f_1 t + \sin 2\pi f_2 t) = 2A\cos 2\pi t(\tfrac{f_1 - f_2}{2})\sin 2\pi t(\tfrac{f_1+f_2}{2}).$$

   The first cosine produces beats: the oscillation is too slow to be interpreted as a pitch, it's just a modulation of intensity. There are two beats per cycle: when cosine equals $+1$ and when it equals -1, so the beat frequency is twice the frequency of the cosine: $f_b = f_1 - f_2$.

3. (10 points) Number of decibels N_d the intensity changed is given by

   $$N_d = 10\log_{10}\tfrac{I_{new}}{I_{old}} = 10\log_{10}\tfrac{A^2_{new}}{A^2_{old}} = 20\log_{10}\tfrac{1}{5} \approx -14.0\,dB.$$

4. (40 points)

(a) (15 points) Let's apply second Newton's law to this slab with area A and thickness dz: $F = ma$, where

$$F = A(P(z) - P(z+dz)) = -A\frac{\partial P}{\partial z}dz,$$

and

$$ma = (\rho A dz)\frac{\partial^2 s}{\partial t^2},$$

so

$$\frac{\partial P}{\partial z} = -\rho\frac{\partial^2 s}{\partial t^2}.$$

(b) (5 points) From the equation above we could derive the wave equation for $s(z,t)$. One of the possible solutions of the wave equation is $s(z,t) = s_0\cos(kz - \omega t)$. The other solutions can use sine instead of cosine or have a phase factor: $s(z,t) = s_0\sin(kz - \omega t + \Delta\phi)$. Any of these answers is good.

(c) (7 points) From the part (a), and assuming $s(z,t) = s_0\cos(kz - \omega t)$,

$$\frac{\partial P}{\partial z} = -\rho\frac{\partial^2 s}{\partial t^2} = \rho\omega^2 s_0\cos(kz - \omega t),$$

so, integrating this, we get

$$P(z,t) = \rho\omega^2 s_0(\tfrac{1}{k})\sin(kz - \omega t) + const = \tfrac{1}{k}\rho\omega^2 s_0\sin(kz - \omega t) + const.$$

where is average pressure value, relative to which we have oscillations. In usual circumstances this constant is equal to atmospheric pressure.

(d) (5 points) Phase difference between sine and cosine is 90°, because $\sin(\alpha + \pi/2) = \cos(\alpha)$. We see that the displacement s is proportional to cosine, while pressure P is proportional to sine of the same argument. Then, phase difference between them is 90°.

(e) (8 points) Velocity of the particles is given by $v = \frac{\partial s}{\partial t} = \omega s_0\sin(kz - \omega t)$. Let's again consider thin slab with thickness dz, density ρ and area A, so the mass of this small slab, moving with constant velocity is $dm = dzA\rho$. The average kinetic energy per unit length will be

$$\langle K\rangle = \frac{1}{L}\int_0^L dm\, v^2/2 = \frac{1}{L}\int_0^L dz\, A\rho v^2/2 = \frac{1}{2L}\int_0^L dz\, A\rho v^2 = \frac{1}{2L}\int_0^L dz\, A\rho\omega^2 s_0^2\sin^2(kz - \omega t).$$

Average of square of a sine is 1/2:

$$\lim_{L\to\infty}\frac{1}{L}\int_0^L dz\ \sin^2(kz - \omega t) = \frac{1}{2},$$

so the average kinetic energy per unit length is

$$\langle K\rangle = \frac{1}{4}A\rho\omega^2 s_0^2.$$

5. (20 points)

   (a) (5 points) Average kinetic energy of the atoms $\langle K \rangle = \frac{1}{2} m_A v_{rmsA}^2 = \frac{3}{2} kT$, so the $v_{rmsA} = \sqrt{\frac{3kT}{m_A}}$.

   (b) (5 points) It doesn't matter how many molecules of gas B are mixed with the gas A; if the gases are at equilibrium, we can still assume that their temperatures are the same and $\langle K_B \rangle = \frac{1}{2} m_B v_{rmsB}^2 = \frac{3}{2} kT$, so the $v_{rmsB} = \sqrt{\frac{3kT}{m_B}}$.

   (c) (5 points) Pressure that a single molecule creates is not very well defined – we will use averaged value of pressure over time, so that we can look at this gas as extremely diluted ideal gas, so $PV = N_a kT$ formula still applies, where N_a is number of atoms: $N_a = 1$ for "gas" B and $N_a = NV$ for gas A, as N – is a number density. The temperatures in case of A and B gases are the same, volumes are the same, so ration of pressures will be given by ratios of the number of molecules: $P_A/P_B = NV$.

   (d) (5 points) We already found average kinetic energies, so the total kinetic energy will be $K_{tot} = \langle K_B \rangle + NV \langle K_A \rangle = (NV + 1)\frac{3}{2} kT$.

6. (25 points) We will assume harmonic pendulum, so the angular displacement is sinusoidal: $\theta = \theta_M \cos \omega t$. The potential energy is given by $E_p = mgl(1 - \cos\theta) \approx mg\theta^2/2$ for small angles θ. The kinetic energy $K = mv^2/2$, where $v = \frac{\partial \theta}{\partial t} l = -l\theta_M \omega \sin \omega t$.

   Averaging potential energy, we get

$$\langle E_p \rangle = mg\tfrac{l}{2} \langle \theta_M^2 \cos^2 \omega t \rangle = mgl\theta_M^2/4.$$

   Averaging kinetic energy,

$$\langle K \rangle = \tfrac{m}{2} \langle l^2 \theta_M^2 \omega^2 \sin^2 \omega t \rangle = m\omega^2 l^2 \theta_M^2/4,$$

   because average of sine or cosine squared is 1/2. Then, using $\omega = \sqrt{g/l}$, we get $\langle K \rangle = \langle E_p \rangle$.