

Quiz # 2 solutions

Solutions by Yuri Kiselev

1. (15 points) Density of the water is higher than density of the ice, so a given volume of the ice melts into a smaller volume of the water, so the water level should go down. During the melting, total mass of the compound doesn't change. Mass of the water cube was $m_{ice} = \rho_{ice}V_{ice} = 0.9 \cdot 5^3 g = m_w = \rho_{water}V_w$, where m_w is a mass of water, produced by melting the ice cube. Level of the water changes by $(V_{total}^{after\ melting} - V_{total}^{before\ melting})/A = (V_w - V_{ice})/A = (m_w/\rho_{water} - V_{ice})/A = (0.9 \cdot 5^3/1 - 5^3)/20 = -0.625\ cm$. The fact that we get negative result means that the water level goes down.

2. (15 points)

- (a) (10 points) The fact that the fully submerged structure is neither sinking nor floating means that the average density of the whole structure is equal to the density of water. Average density is given by

$$\rho = \frac{m}{V} = \frac{m_{ice} + m_{metal}}{V_{ice} + V_{metal}} = \frac{\rho_{ice}V_{ice} + \rho_{metal}V_{metal}}{V_{ice} + V_{metal}} = \rho_{water}.$$

Solving this equation, we find the volume of the metal:

$$V_{metal} = \frac{\rho_{ice}V_{ice} - \rho_{water}V_{ice}}{\rho_{water} - \rho_{metal}} = \frac{0.9 \cdot 5^3 - 1 \cdot 5^3}{1 - 7.8} \approx 1.84\ cm^3.$$

- (b) (5 points) The shape of the metal doesn't matter – it does not enter into the equations above. Also, the buoyancy force depends only on the submerged volume, not on its shape.
3. (15 points) Pressure inside the liquid on the same height level must be the same. Let's look at the level just below water – gauge pressure (relative to atmospheric) is $P_{h2} = \rho_{water}gh_2$. At the same level in the right tube pressure equals to $P_{h1} = \rho_{mercury}gh_1$. They should be equal, so $\rho_{water}gh_2 = \rho_{mercury}gh_1$ and $h_2/h_1 = \rho_{mercury}/\rho_{water}$.
4. (15 points) This is an incorrect argument, because not all forces are taken into account. In addition to the force due to the atmospheric pressure there are also gravitational and normal forces due to a reaction from bottom of the container and the sides of the container.

Consider forces acting on the water inside the left and right parts of the container. Forces, that have vertical component, which act on the water in the left part of the container are: gravitational force, normal forces from the diagonal sides and a force, associated with water pressure at the bottom. Same for the right part, but vertical

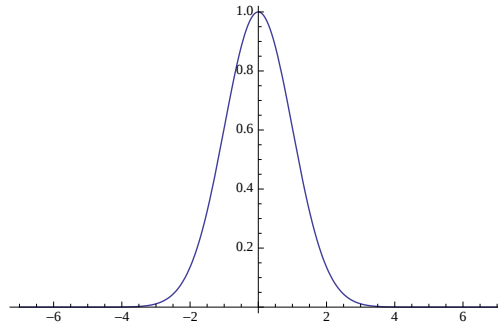


Figure 1: $y(x, t)$ for $t = 0$, $a = 1$, $y_m = 1$.

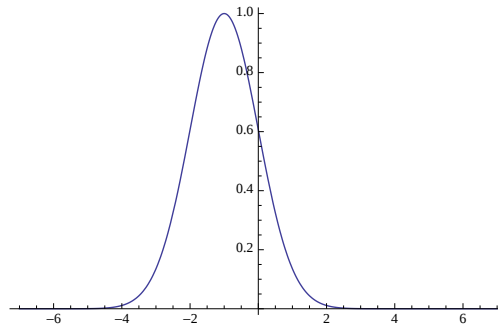


Figure 2: $y(x, t)$ for $t = 1$, $a = 1$, $y_m = 1$, $v = 1$ m/s.

component of the normal forces for the right part is zero, because the normal force is horizontal. So, $m_L g = N_y + P_L A$ for the left part and $m_R g = 0 + P_R A$ for the right one. The difference in gravitational force will be compensated by the normal force component, but pressure at the bottom will be the same.

5. (15 points)

- (a) (4 points) No, it's not, see Fig. 1. The periodic wave is such that $y(x) = y(x + n\Delta x)$ for some Δx and any integer n .
- (b) (4 points) This is a traveling wave: the position of maximum is given by $x_{max} + vt = 0$ (and $y = y_m$), so $x_{max} = -vt$. So, position of the maximum moves: the wave is traveling. See Fig. 2 for an idea what happens with the wave after 1 second, $v = 1$ m/s.
- (c) (4 points) As we discussed above, the position of the maximum of the wave is given by $x_{max} = -vt$. So, the wave is traveling to the left with velocity v .
- (d) (3 points) The wave is not damped – it keeps its shape in different times (compare Fig. 1 and Fig. 2). It only moves as a whole to the left.

6. (25 points)

- (a) (17 points) Tension of the string holds the hanging mass, so $F_T = mg$. The velocity of waves on the string is given by $v = \sqrt{\frac{F_T}{\mu}}$. The lowest frequency corresponds to the fundamental harmonic: length of the vibrating string $l = \lambda/2 = vT/2 = v/(2f)$. Then, minimum resonant frequency is $f = v/(2l) = \frac{1}{2l} \sqrt{\frac{mg}{\mu}}$.
- (b) (8 points) Fundamental is the same as first harmonic and first overtone is a second harmonic: $l = \lambda$ instead of $l = \lambda/2$ above. Then, $l = \lambda = vT = v/f$ and so frequency is $f = v/l = \frac{1}{l} \sqrt{\frac{mg}{\mu}}$. It's two times higher than the fundamental one.
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