4B QUIZZ #1

This Quiz is closed book and no notes and no crib sheet. Calculators are ok but they should not contain physics equations nor text in their memory. Make sure to always define an appropriate coordinate system and indicate its origin and positive directions. Make large and neat figures. Specify the units of numerical answers.

PROBLEM 1 (40 points)

A uniform bar of mass \( M \) and length \( \ell \) is laying on a table as shown in the Figure. It sticks out beyond the edge of the table by a length \( x \). At its end a small mass \( m \) is positioned as shown.

a. Calculate the maximum value of \( x \) such that the bar is just about but not quite starting to fall.

b. The bar is moved to the left so that only a length \( x/2 \) sticks out beyond the edge of the table. Calculate the inertial moment of the bar alone relative to the edge of the table.

c. Calculate the angular acceleration at the very instant that the bar starts to move.

![Diagram of the bar and small mass](image)
PROBLEM 2 (30 points).

A mass $m$ is suspended by two identical springs as shown in the Figure. The spring above the mass has a spring constant $k_1$ while the spring below the mass has a spring constant $k_2$.

a. Derive the DE for the motion of the mass.

b. What is the order of the DE? Is it linear? Is it homogeneous?

c. Find the most general solution of the DE in a).

d. Specify as many constants as possible in the solution. Can all constants be determined? If not, why not?

PROBLEM 3 (30 points).

A mass $m$ is positioned on a horizontal surface without friction. A spring with spring constant $k$ is attached to it as shown in the Figure. Initially the spring is neither stretched nor compressed. Someone applies a brief impulse to the mass so that it leaves its position with velocity $v_0$ to the left. The mass starts an oscillation without hitting the wall.

a. Determine the differential equation.

b. What is the frequency of the oscillation?

c. What is the amplitude of the oscillation?
d. Is the oscillation damped?

PROBLEM 4 (Extra credit, 30 points).

A pendulum is suspended from the ceiling of a railroad car. Its length is $\ell$ and the object at its end has a mass $m$. The pendulum is made to oscillate. Of course its natural angular frequency is given by $\omega = \sqrt{g/\ell}$.

Suppose the train is moving at a constant velocity $v_0$ down the rails. With what frequency will the pendulum now oscillate? Explain.