CHAPTER 15: Wave Motion

Responses to Questions

5. The speed of sound in air obeys the equation $v = \sqrt{B/\rho}$. If the bulk modulus is approximately constant and the density of air decreases with temperature, then the speed of sound in air should increase with increasing temperature.

6. First, estimate the number of wave crests that pass a given point per second. This is the frequency of the wave. Then, estimate the distance between two successive crests, which is the wavelength. The product of the frequency and the wavelength is the speed of the wave.

$\boxed{7.}$ The speed of sound is defined as $v = \sqrt{B/\rho}$, where *B* is the bulk modulus and *ρ* is the density of the material. The bulk modulus of most solids is at least 10^6 times as great as the bulk modulus of air. This difference overcomes the larger density of most solids, and accounts for the greater speed of sound in most solids than in air.

10. Yes. Any function of (*x* - *vt*) will represent wave motion because it will satisfy the wave equation, Eq. 15-16.

18. AM radio waves have a much longer wavelength than FM radio waves. How much waves bend, or diffract, around obstacles depends on the wavelength of the wave in comparison to the size of the obstacle. A hill is much larger than the wavelength of FM waves, and so there will be a “shadow” region behind the hill. However, the hill is not large compared to the wavelength of AM signals, so the AM radio waves will bend around the hill.

Solutions to Problems

3. The elastic and bulk moduli are taken from Table 12-1. The densities are taken from Table 13-1.

(*a*) For water: $$v = \sqrt{B/\rho} = \sqrt{\frac{2.0\times10^9\ \mathrm{N/m^2}}{1.00\times10^3\ \mathrm{kg/m^3}}} = \boxed{1400\ \mathrm{m/s}}$$

(*b*) For granite: $$v = \sqrt{E/\rho} = \sqrt{\frac{45\times10^9\ \mathrm{N/m^2}}{2.7\times10^3\ \mathrm{kg/m^3}}} = \boxed{4100\ \mathrm{m/s}}$$

(*c*) For steel: $$v = \sqrt{E/\rho} = \sqrt{\frac{200\times10^9\ \mathrm{N/m^2}}{7.8\times10^3\ \mathrm{kg/m^3}}} = \boxed{5100\ \mathrm{m/s}}$$

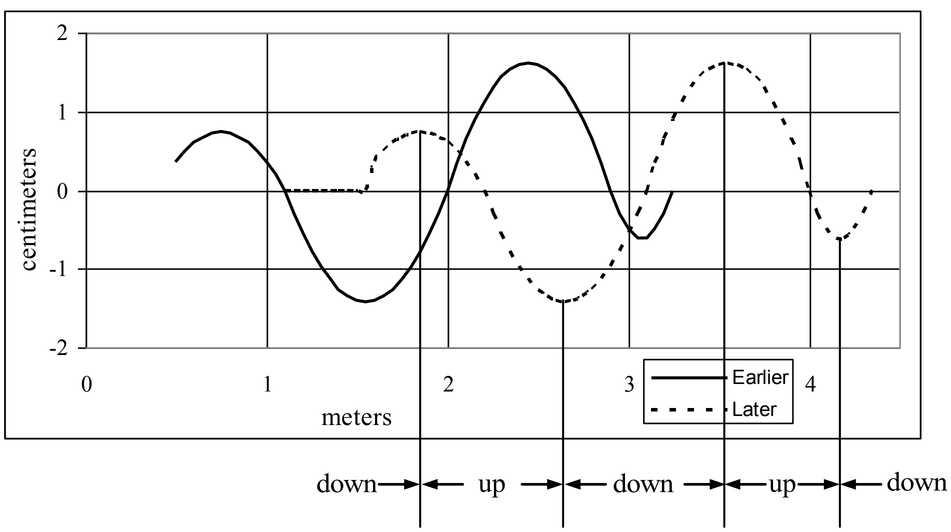
6. To find the time for a pulse to travel from one end of the cord to the other, the velocity of the pulse on the cord must be known. For a cord under tension, we have Eq. 15-2, $v=\sqrt{F_T/\mu}$.

$$v=\frac{\Delta x}{\Delta t}=\sqrt{\frac{F_T}{\mu}} \rightarrow \Delta t=\frac{\Delta x}{\sqrt{\frac{F_T}{\mu}}}=\frac{8.0\,\text{m}}{\sqrt{\frac{140\,\text{N}}{(0.65\,\text{kg})/(8.0\,\text{m})}}}=\boxed{0.19\,\text{s}}$$

8. The speed of the water wave is given by $v=\sqrt{B/\rho}$, where B is the bulk modulus of water, from Table 12-1, and ρ is the density of sea water, from Table 13-1. The wave travels twice the depth of the ocean during the elapsed time.

$$v=\frac{2l}{t} \rightarrow l=\frac{vt}{2}=\frac{t}{2}\sqrt{\frac{B}{\rho}}=\frac{2.8\,\text{s}}{2}\sqrt{\frac{2.0\times10^9\,\text{N/m}^2}{1.025\times10^3\,\text{kg/m}^3}}=\boxed{2.0\times10^3\,\text{m}}$$

11. (*a*) The shape will not change. The wave will move 1.10 meters to the right in 1.00 seconds. See the graph. The parts of the string that are moving up or down are indicated.



(*b*) At the instant shown, the string at point A will be moving down. As the wave moves to the right, the string at point A will move down by 1 cm in the time it takes the "valley" between 1 m and 2 m to move to the right by about 0.25 m.

$$v = \frac{\Delta y}{\Delta t} = \frac{-1\,\text{cm}}{0.25\,\text{m}/1.10\,\text{m/s}} \approx \boxed{-4\,\text{cm/s}}$$

This answer will vary depending on the values read from the graph.

12. We assume that the wave will be transverse. The speed is given by Eq. 15-2. The tension in the wire is equal to the weight of the hanging mass. The linear mass density is the volume mass density times the cross-sectional area of the wire. The volume mass density is found in Table 13-1.

$$v = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{m_{\text{ball}}g}{\frac{\rho V}{l}}} = \sqrt{\frac{m_{\text{ball}}g}{\rho\frac{Al}{l}}} = \sqrt{\frac{(5.0\,\text{kg})(9.80\,\text{m/s}^2)}{(7800\,\text{kg/m}^3)\pi(0.50\times10^{-3}\,\text{m})^2}} = \boxed{89\,\text{m/s}}$$

15. From Eq. 15-7, if the speed, medium density, and frequency of the two waves are the same, then the intensity is proportional to the square of the amplitude.

$$I_2/I_1 = E_2/E_1 = A_2^2/A_1^2 = 3 \rightarrow A_2/A_1 = \sqrt{3} = \boxed{1.73}$$

The more energetic wave has the larger amplitude.

16. (*a*) Assume that the earthquake waves spread out spherically from the source. Under those

conditions, Eq. (15-8ab) applies, stating that intensity is inversely proportional to the square of the distance from the source of the wave.

$$I_{45\,\text{km}} / I_{15\,\text{km}} = (15\,\text{km})^2 / (45\,\text{km})^2 = \boxed{0.11}$$

(*b*) The intensity is proportional to the square of the amplitude, and so the amplitude is inversely proportional to the distance from the source of the wave.

$$A_{45\,\text{km}} / A_{15\,\text{km}} = 15\,\text{km}/45\,\text{km} = \boxed{0.33}$$

22. (a) The only difference is the direction of motion.

$$D(x,t) = 0.015\sin(25x + 1200t)$$

(*b*) The speed is found from the wave number and the angular frequency, Eq. 15-12.

$$v = \frac{\omega}{k} = \frac{1200\,\text{rad/s}}{25\,\text{rad/m}} = \boxed{48\,\text{m/s}}$$

24. The traveling wave is given by $D = 0.22\sin(5.6x + 34t)$.

(*a*) The wavelength is found from the coefficient of *x*.

$$5.6\,\text{m}^{-1} = \frac{2\pi}{\lambda} \quad \rightarrow \quad \lambda = \frac{2\pi}{5.6\,\text{m}^{-1}} = 1.122\,\text{m} \approx \boxed{1.1\,\text{m}}$$

(*b*) The frequency is found from the coefficient of *t*.

$$34\,\text{s}^{-1} = 2\pi f \quad \rightarrow \quad f = \frac{34\,\text{s}^{-1}}{2\pi} = 5.411\,\text{Hz} \approx \boxed{5.4\,\text{Hz}}$$

(*c*) The velocity is the ratio of the coefficients of *t* and *x*.

$$v = \lambda f = \frac{2\pi}{5.6\,\text{m}^{-1}} \frac{34\,\text{s}^{-1}}{2\pi} = 6.071\,\text{m/s} \approx \boxed{6.1\,\text{m/s}}$$

Because both coefficients are positive, the velocity is in the $\boxed{\text{negative } x \text{ direction}}$.

(*d*) The amplitude is the coefficient of the sine function, and so is $\boxed{0.22\,\text{m}}$.

(*e*) The particles on the cord move in simple harmonic motion with the same frequency as the wave. From Chapter 14, $v_{\max} = D\omega = 2\pi fD.$

$$v_{\max} = 2\pi fD = 2\pi\left(\frac{34\,\text{s}^{-1}}{2\pi}\right)(0.22\,\text{m}) = \boxed{7.5\,\text{m/s}}$$

The minimum speed is when a particle is at a turning point of its motion, at which time the speed is 0.

$$v_{\min} = \boxed{0}$$

26. The displacement of a point on the cord is given by the wave, $D(x,t) = 0.12\sin(3.0x - 15.0t)$. The velocity of a point on the cord is given by $\dfrac{\partial D}{\partial t}$.

$$D(0.60\,\text{m}, 0.20\,\text{s}) = (0.12\,\text{m})\sin\left[\left(3.0\text{m}^{-1}\right)(0.60\,\text{m}) - \left(15.0\,\text{s}^{-1}\right)(0.20\,\text{s})\right] = \boxed{-0.11\,\text{m}}$$

$$\frac{\partial D}{\partial t} = (0.12\,\text{m})\left(-15.0\,\text{s}^{-1}\right)\cos(3.0x - 15.0t)$$

$$\frac{\partial D}{\partial t}(0.60\,\text{m}, 0.20\,\text{s}) = (0.12\,\text{m})\left(-15.0\,\text{s}^{-1}\right)\cos\left[\left(3.0\text{m}^{-1}\right)(0.60\,\text{m}) - \left(15.0\,\text{s}^{-1}\right)(0.20\,\text{s})\right] = \boxed{-0.65\,\text{m/s}}$$

28. (*a*) The wavelength is the speed divided by the frequency.

$$\lambda = \frac{v}{f} = \frac{345\,\text{m/s}}{524\,\text{Hz}} = \boxed{0.658\,\text{m}}$$

(*b*) In general, the phase change in degrees due to a time difference is given by $\dfrac{\Delta\phi}{360^\circ} = \dfrac{\Delta t}{T}$.

$$\frac{\Delta\phi}{360^\circ} = \frac{\Delta t}{T} = f\,\Delta t \;\rightarrow\; \Delta t = \frac{1}{f}\frac{\Delta\phi}{360^\circ} = \frac{1}{524\,\text{Hz}}\left(\frac{90^\circ}{360^\circ}\right) = \boxed{4.77\times10^{-4}\,\text{s}}$$

(*c*) In general, the phase change in degrees due to a position difference is given by $\frac{\Delta\phi}{360°}=\frac{\Delta x}{\lambda}$.

$$\frac{\Delta\phi}{360°}=\frac{\Delta x}{\lambda} \quad\rightarrow\quad \Delta\phi=\frac{\Delta x}{\lambda}(360°)=\frac{0.044\,\mathrm{m}}{0.658\,\mathrm{m}}(360°)=\boxed{24.1°}$$

29. The amplitude is 0.020 cm, the wavelength is 0.658 m, and the frequency is 524 Hz. The displacement is at its most negative value at *x* = 0, *t* = 0, and so the wave can be represented by a cosine that is phase shifted by half of a cycle.

$$D(x,t)=A\cos(kx-\omega t+\phi)$$

$$A=0.020\,\mathrm{cm}\ ;\ k=\frac{2\pi}{\lambda}=\frac{2\pi f}{v}=\frac{2\pi(524\,\mathrm{Hz})}{345\,\mathrm{m/s}}=9.54\,\mathrm{m}^{-1}\ ;\ \omega=2\pi f=2\pi(524\,\mathrm{Hz})=3290\,\mathrm{rad/s}$$

$$\boxed{D(x,t)=(0.020\,\mathrm{cm})\cos\left[\left(9.54\,\mathrm{m}^{-1}\right)x-(3290\,\mathrm{rad/s})t+\pi\right],\ x\text{ in m, }t\text{ in s}}$$

Other equivalent expressions include the following.

$$D(x,t)=-(0.020\,\mathrm{cm})\cos\left[\left(9.54\,\mathrm{m}^{-1}\right)x-(3290\,\mathrm{rad/s})t\right]$$

$$D(x,t)=(0.020\,\mathrm{cm})\sin\left[\left(9.54\,\mathrm{m}^{-1}\right)x-(3290\,\mathrm{rad/s})t+\tfrac{3}{2}\pi\right]$$

32. To be a solution of the wave equation, the function must satisfy Eq. 15-16, $\frac{\partial^2 D}{\partial x^2}=\frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$.

(*a*) $D=A\ln(x+vt)$

$$\frac{\partial D}{\partial x}=\frac{A}{x+vt}\ ;\ \frac{\partial^2 D}{\partial x^2}=-\frac{A}{(x+vt)^2}\ ;\ \frac{\partial D}{\partial t}=\frac{Av}{x+vt}\ ;\ \frac{\partial^2 D}{\partial t^2}=-\frac{Av^2}{(x+vt)^2}$$

This gives $\frac{\partial^2 D}{\partial x^2}=\frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$, and so $\boxed{\text{yes, the function is a solution.}}$

(*b*) $D=(x-vt)^4$

$$\frac{\partial D}{\partial x}=4(x-vt)^3\ ;\ \frac{\partial^2 D}{\partial x^2}=12(x-vt)^2\ ;\ \frac{\partial D}{\partial t}=-4v(x-vt)^3\ ;\ \frac{\partial^2 D}{\partial t^2}=12v^2(x-vt)^2$$

This gives $\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$, and so $\boxed{\text{yes, the function is a solution.}}$

34. Find the various derivatives for the linear combination.

$$D(x,t) = C_1 D_1 + C_2 D_2 = C_1 f_1(x,t) + C_2 f_2(x,t)$$
$$\frac{\partial D}{\partial x} = C_1\frac{\partial f_1}{\partial x} + C_2\frac{\partial f_2}{\partial x} \ ; \ \frac{\partial^2 D}{\partial x^2} = C_1\frac{\partial^2 f_1}{\partial x^2} + C_2\frac{\partial^2 f_2}{\partial x^2}$$
$$\frac{\partial D}{\partial t} = C_1\frac{\partial f_1}{\partial t} + C_2\frac{\partial f_2}{\partial t} \ ; \ \frac{\partial^2 D}{\partial t^2} = C_1\frac{\partial^2 f_1}{\partial t^2} + C_2\frac{\partial^2 f_2}{\partial t^2}$$

To satisfy the wave equation, we must have $\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$. Use the fact that both f_1 and f_2 satisfy the wave equation.

$$\frac{\partial^2 D}{\partial x^2} = C_1\frac{\partial^2 f_1}{\partial x^2} + C_2\frac{\partial^2 f_2}{\partial x^2} = C_1\left[\frac{1}{v^2}\frac{\partial^2 f_1}{\partial t^2}\right] + C_2\left[\frac{1}{v^2}\frac{\partial^2 f_2}{\partial t^2}\right] = \frac{1}{v^2}\left[C_1\frac{\partial^2 f_1}{\partial t^2} + C_2\frac{\partial^2 f_2}{\partial t^2}\right] = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$$

Thus we see that $\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$, and so *D* satisfies the wave equation.

$\boxed{37.}$ (*a*) For the wave in the lighter cord, $D(x,t) = (0.050\,\text{m})\sin\left[(7.5\,\text{m}^{-1})x - (12.0\,\text{s}^{-1})t\right]$.

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{(7.5\,\text{m}^{-1})} = \boxed{0.84\,\text{m}}$$

(*b*) The tension is found from the velocity, using Eq. 15-2.

$$v = \sqrt{\frac{F_\text{T}}{\mu}} \ \rightarrow \ F_\text{T} = \mu v^2 = \mu\frac{\omega^2}{k^2} = (0.10\,\text{kg/m})\frac{(12.0\,\text{s}^{-1})^2}{(7.5\,\text{m}^{-1})^2} = \boxed{0.26\,\text{N}}$$

(*c*) The tension and the frequency do not change from one section to the other.

$$F_{\text{T}1} = F_{\text{T}2} \ \rightarrow \ = \mu_1\frac{\omega_1^2}{k_1^2} = \mu_2\frac{\omega_2^2}{k_2^2} \ \rightarrow \ \lambda_2 = \lambda_1\sqrt{\frac{\mu_1}{\mu_2}} = \frac{2\pi}{k_1}\sqrt{\frac{\mu_1}{\mu_2}} = \frac{2\pi}{(7.5\,\text{m}^{-1})}\sqrt{0.5} = \boxed{0.59\,\text{m}}$$

42. (*a*) The resultant wave is the algebraic sum of the two component waves.

$$\begin{aligned} D &= D_1 + D_2 = A\sin(kx-\omega t) + A\sin(kx-\omega t+\phi) = A\left[\sin(kx-\omega t) + A\sin(kx-\omega t+\phi)\right] \\ &= A\left\{2\sin\tfrac{1}{2}\left[(kx-\omega t)+(kx-\omega t+\phi)\right]\right\}\left\{\cos\tfrac{1}{2}\left[(kx-\omega t)-(kx-\omega t+\phi)\right]\right\} \\ &= 2A\left\{\sin\tfrac{1}{2}(2kx-2\omega t+\phi)\right\}\left\{\cos\tfrac{1}{2}(\phi)\right\} = \boxed{\left(2A\cos\frac{\phi}{2}\right)\sin\left(kx-\omega t+\frac{\phi}{2}\right)} \end{aligned}$$

(*b*) The amplitude is the absolute value of the coefficient of the sine function, $\boxed{\left|2A\cos\frac{\phi}{2}\right|}$. The wave is $\boxed{\text{purely sinusoidal}}$ because the dependence on *x* and *t* is $\sin\left(kx-\omega t+\frac{\phi}{2}\right)$.

(*c*) If $\phi = 0, 2\pi, 4\pi, \ldots, 2n\pi$, then the amplitude is

$$\left|2A\cos\frac{\phi}{2}\right| = \left|2A\cos\frac{2n\pi}{2}\right| = \left|2A\cos n\pi\right| = \left|2A(\pm 1)\right|$$

$=2A$, which is constructive interference. If $\phi = \pi, 3\pi, 5\pi, \ldots, (2n+1)\pi$, then the amplitude is $\left|2A\cos\frac{\phi}{2}\right| = \left|2A\cos\frac{(2n+1)\pi}{2}\right| = \left|2A\cos\left[\left(n+\tfrac{1}{2}\right)\pi\right]\right| = 0$, which is destructive interference.

(*d*) If $\phi = \frac{\pi}{2}$, then the resultant wave is as follows.

$$D = \left(2A\cos\frac{\phi}{2}\right)\sin\left(kx-\omega t+\frac{\phi}{2}\right) = \left(2A\cos\frac{\pi}{4}\right)\sin\left(kx-\omega t+\frac{\pi}{4}\right) = \sqrt{2}A\sin\left(kx-\omega t+\frac{\pi}{4}\right)$$

This wave has an amplitude of $\sqrt{2}A$, is traveling in the positive *x* direction, and is shifted to the left by an eighth of a cycle. This is "halfway" between the two original waves. The displacement is $\frac{1}{2}A$ at the origin at *t* = 0.

45. The oscillation corresponds to the fundamental. The frequency of that oscillation is $f_1 = \frac{1}{T} = \frac{1}{1.5\,\text{s}} = \frac{2}{3}\,\text{Hz}$. The bridge, with both ends fixed, is similar to a vibrating string, and so

$\boxed{f_n = nf_1 = \frac{2n}{3}\,\text{Hz},\ n = 1, 2, 3\text{K}}$. The periods are the reciprocals of the frequency, and so

$\boxed{T_n = \frac{1.5\,\text{s}}{n},\ n = 1, 2, 3\text{K}}$.

48. Adjacent nodes are separated by a half-wavelength, as examination of Figure 15-26 will show.

$$\lambda = \frac{v}{f} \quad \rightarrow \quad \Delta x_{\text{node}} = \tfrac{1}{2}\lambda = \frac{v}{2f} = \frac{96\,\text{m/s}}{2(445\,\text{Hz})} = \boxed{0.11\,\text{m}}$$

51. The speed of the wave is given by Eq. 15-2, $v = \sqrt{F_T/\mu}$. The wavelength of the fundamental is $\lambda_1 = 2l$. Thus the frequency of the fundamental is $f_1 = \frac{v}{\lambda_1} = \frac{1}{2l}\sqrt{\frac{F_T}{\mu}}$. Each harmonic is present in a vibrating string, and so $f_n = nf_1 = \boxed{\frac{n}{2l}\sqrt{\frac{F_T}{\mu}}},\ n = 1, 2, 3\text{K}$.

54. The standing wave is given by $D = (2.4\,\text{cm})\sin(0.60x)\cos(42t)$.

(*a*) The distance between nodes is half of a wavelength.

$$d = \tfrac{1}{2}\lambda = \tfrac{1}{2}\frac{2\pi}{k} = \frac{\pi}{0.60\,\text{cm}^{-1}} = 5.236\,\text{cm} \approx \boxed{5.2\,\text{cm}}$$

(*b*) The component waves travel in opposite directions. Each has the same frequency and speed, and each has half the amplitude of the standing wave.

$$A = \tfrac{1}{2}(2.4\,\text{cm}) = \boxed{1.2\,\text{cm}}\ ;\ f = \frac{\omega}{2\pi} = \frac{42\,\text{s}^{-1}}{2\pi} = 6.685\,\text{Hz} \approx \boxed{6.7\,\text{Hz}}\ ;$$
$$v = \lambda f = 2d_{\text{node}}f = 2(5.236\,\text{cm})(6.685\,\text{Hz}) = 70.01\,\text{cm/s} \approx \boxed{70\,\text{cm/s}}\ (2\text{ sig. fig.})$$

(*c*) The speed of a particle is given by $\frac{\partial D}{\partial t}$.

$$\frac{\partial D}{\partial t} = \frac{\partial}{\partial t}\left[(2.4\,\text{cm})\sin(0.60x)\cos(42t)\right] = (-42\,\text{rad/s})(2.4\,\text{cm})\sin(0.60x)\sin(42t)$$

$$\frac{\partial D}{\partial t}(3.20\,\text{cm},\ 2.5\text{s}) = (-42\,\text{rad/s})(2.4\,\text{cm})\sin\left[\left(0.60\,\text{cm}^{-1}\right)(3.20\,\text{cm})\right]\sin\left[(42\,\text{rad/s})(2.5\,\text{s})\right]$$

$$= \boxed{92\,\text{cm/s}}$$

$\boxed{55.}$ (*a*) The given wave is $D_1 = 4.2\sin(0.84x - 47t + 2.1)$. To produce a standing wave, we simply need to add a wave of the same characteristics but traveling in the opposite direction. This is the appropriate wave.

$$\boxed{D_2 = 4.2\sin(0.84x + 47t + 2.1)}$$

(*b*) The standing wave is the sum of the two component waves. We use the trigonometric identity that $\sin\theta_1 + \sin\theta_2 = 2\sin\frac{1}{2}(\theta_1 + \theta_2)\cos\frac{1}{2}(\theta_1 - \theta_2)$.

$$\begin{aligned} D = D_1 + D_2 &= 4.2\sin(0.84x - 47t + 2.1) + 4.2\sin(0.84x + 47t + 2.1) \\ &= 4.2(2)\left\{\sin\tfrac{1}{2}\left[(0.84x - 47t + 2.1) + (0.84x + 47t + 2.1)\right]\right\} \\ &\qquad \left\{\cos\tfrac{1}{2}\left[(0.84x - 47t + 2.1) - (0.84x + 47t + 2.1)\right]\right\} \\ &= 8.4\sin(0.84x + 2.1)\cos(-47t) = \boxed{8.4\sin(0.84x + 2.1)\cos(47t)} \end{aligned}$$

We note that the origin is NOT a node.

60. (*a*) The maximum swing is twice the amplitude of the standing wave. Three loops is 1.5 wavelengths, and the frequency is given.

$$A = \tfrac{1}{2}(8.00\,\text{cm}) = 4.00\,\text{cm} \ ;\ \omega = 2\pi f = 2\pi(120\,\text{Hz}) = 750\,\text{rad/s} \ ;$$

$$k = \frac{2\pi}{\lambda} \ \rightarrow \ ;\ \tfrac{3}{2}\lambda = 1.64\,\text{m} \ \rightarrow \ \lambda = 1.09\,\text{m} \ ;\ k = \frac{2\pi}{1.09\,\text{m}} = 5.75\,\text{m}^{-1}$$

$$D = A\sin(kx)\cos(\omega t) = \boxed{(4.00\,\text{cm})\sin\left[\left(5.75\,\text{m}^{-1}\right)x\right]\cos\left[(750\,\text{rad/s})t\right]}$$

(*b*) Each component wave has the same wavelength, the same frequency, and half the amplitude of the standing wave.

$$\boxed{\begin{array}{l} D_1 = (2.00\,\text{cm})\sin\left[\left(5.75\,\text{m}^{-1}\right)x - (750\,\text{rad/s})t\right] \\ D_2 = (2.00\,\text{cm})\sin\left[\left(5.75\,\text{m}^{-1}\right)x + (750\,\text{rad/s})t\right] \end{array}}$$

63. The standing wave is the sum of the two individual standing waves. We use the trigonometric identities for the cosine of a difference and a sum.

$$\cos(\theta_1 - \theta_2) = \cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2\ ;\ \cos(\theta_1 + \theta_2) = \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2$$

$$\begin{aligned} D = D_1 + D_2 &= A\cos(kx - \omega t) + A\cos(kx + \omega t) = A\left[\cos(kx - \omega t) + \cos(kx + \omega t)\right] \\ &= A\left[\cos kx\cos\omega t + \sin kx\sin\omega t + \cos kx\cos\omega t - \sin kx\sin\omega t\right] \\ &= 2A\cos kx\cos\omega t \end{aligned}$$

Thus the standing wave is $D = 2A\cos kx\cos\omega t$. The nodes occur where the position term forces $D = 2A\cos kx\cos\omega t = 0$ for all time. Thus $\cos kx = 0 \rightarrow kx = \pm(2n+1)\frac{\pi}{2},\ n = 0,1,2,\text{L}$. Thus, since $k = 2.0\,\text{m}^{-1}$, we have $\boxed{x = \pm\left(n + \tfrac{1}{2}\right)\frac{\pi}{2}\,\text{m},\ n = 0,1,2,\text{L}}$.

65. The speed in the second medium can be found from the law of refraction, Eq. 15-19.

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1} \rightarrow v_2 = v_1\frac{\sin\theta_2}{\sin\theta_1} = (8.0\,\text{km/s})\left(\frac{\sin 31^\circ}{\sin 52^\circ}\right) = \boxed{5.2\,\text{km/s}}$$

$\boxed{67.}$ The angle of refraction can be found from the law of refraction, Eq. 15-19. The relative velocities can be found from the relationship given in the problem.

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1} = \frac{331 + 0.60T_2}{331 + 0.60T_1} \rightarrow \sin\theta_2 = \sin 33^\circ\frac{331 + 0.60(-15)}{331 + 0.60(25)} = \sin 33^\circ\frac{322}{346} = 0.5069$$

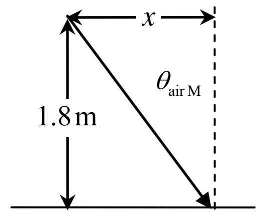
$$\theta_2 = \sin^{-1}0.5069 = \boxed{30^\circ}\ (2\text{ sig. fig.})$$

68. (*a*) Eq. 15-19 gives the relationship between the angles and the speed of sound in the two media.

For total internal reflection (for no sound to enter the water), $\theta_{\text{water}} = 90^\circ$ or $\sin\theta_{\text{water}} = 1$. The air is the "incident" media. Thus the incident angle is given by the following.

$$\frac{\sin\theta_{\text{air}}}{\sin\theta_{\text{water}}}=\frac{v_{\text{air}}}{v_{\text{water}}}\ ;\ \theta_{\text{air}}=\theta_{\text{i}}=\sin^{-1}\left[\sin\theta_{\text{water}}\frac{v_{\text{air}}}{v_{\text{water}}}\right]\ \rightarrow\ \boxed{\theta_{\text{iM}}=\sin^{-1}\left[\frac{v_{\text{air}}}{v_{\text{water}}}\right]=\sin^{-1}\left[\frac{v_{\text{i}}}{v_{\text{r}}}\right]}$$

(*b*) From the angle of incidence, the distance is found. See the diagram.



$$\theta_{\text{air M}}=\sin^{-1}\frac{v_{\text{air}}}{v_{\text{water}}}=\sin^{-1}\frac{343\,\text{m/s}}{1440\,\text{m/s}}=13.8^\circ$$

$$\tan\theta_{\text{air M}}=\frac{x}{1.8\,\text{m}}\ \rightarrow\ x=(1.8\,\text{m})\tan 13.8^\circ=\boxed{0.44\,\text{m}}$$