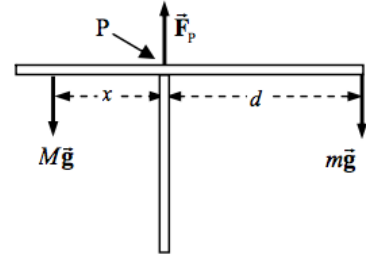


Chapter 12 HW Solutions

4. (a) See the free-body diagram. Calculate torques about the pivot point P labeled in the diagram. The upward force at the pivot will not have any torque. The total torque is zero since the crane is in equilibrium.

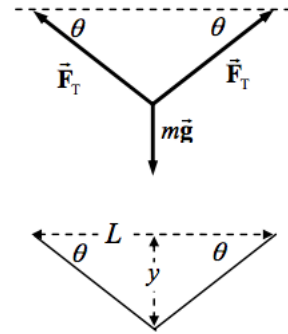
$$\sum \tau = Mgx - mgd = 0 \rightarrow x = \frac{md}{M} = \frac{(2800 \text{ kg})(7.7 \text{ m})}{(9500 \text{ kg})} = \boxed{2.3 \text{ m}}$$



- (b) Again we sum torques about the pivot point. Mass m is the unknown in this case, and the counterweight is at its maximum distance from the pivot.

$$\sum \tau = Mgx_{\max} - m_{\max}gd = 0 \rightarrow m_{\max} = \frac{Mx_{\max}}{d} = \frac{(9500 \text{ kg})(3.4 \text{ m})}{(7.7 \text{ m})} = \boxed{4200 \text{ kg}}$$

7. Since the backpack is midway between the two trees, the angles in the diagram are equal. Write Newton's second law for the vertical direction for the point at which the backpack is attached to the cord, with the weight of the backpack being the downward vertical force. The angle is determined by the distance between the trees and the amount of sag at the midpoint, as illustrated in the second diagram.



$$(a) \theta = \tan^{-1} \frac{y}{L/2} = \tan^{-1} \frac{1.5 \text{ m}}{3.3 \text{ m}} = 24.4^\circ$$

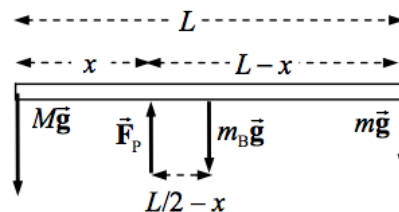
$$\sum F_y = 2F_T \sin \theta_1 - mg = 0 \rightarrow$$

$$F_T = \frac{mg}{2 \sin \theta_1} = \frac{(19 \text{ kg})(9.80 \text{ m/s}^2)}{2 \sin 24.4^\circ} = 225.4 \text{ N} \approx \boxed{230 \text{ N}}$$

$$(b) \theta = \tan^{-1} \frac{y}{L/2} = \tan^{-1} \frac{0.15 \text{ m}}{3.3 \text{ m}} = 2.60^\circ$$

$$F_T = \frac{mg}{2 \sin \theta_1} = \frac{(19 \text{ kg})(9.80 \text{ m/s}^2)}{2 \sin 2.60^\circ} = 2052 \text{ N} \approx \boxed{2100 \text{ N}}$$

10. The pivot should be placed so that the net torque on the board is zero. We calculate torques about the pivot point, with counterclockwise torques positive. The upward force \vec{F}_p at the pivot point is shown, but it exerts no torque about the pivot point. The mass of the child is m , the mass of the adult is M , the mass of the board is m_B , and the center of gravity is at the middle of the board.



(a) Ignore the force $m_B g$.

$$\sum \tau = Mg x - mg(L - x) = 0 \rightarrow$$

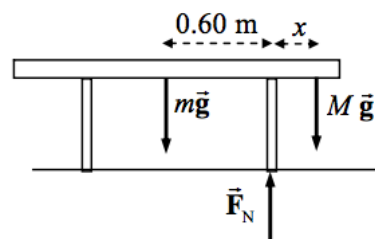
$$x = \frac{m}{m + M} L = \frac{(25 \text{ kg})}{(25 \text{ kg} + 75 \text{ kg})} (9.0 \text{ m}) = 2.25 \text{ m} \approx \boxed{2.3 \text{ m from adult}}$$

(b) Include the force $m_B g$.

$$\sum \tau = Mg x - mg(L - x) - m_B g(L/2 - x) = 0$$

$$x = \frac{(m + m_B/2)}{(M + m + m_B)} L = \frac{(25 \text{ kg} + 7.5 \text{ kg})}{(75 \text{ kg} + 25 \text{ kg} + 15 \text{ kg})} (9.0 \text{ m}) = 2.54 \text{ m} \approx \boxed{2.5 \text{ m from adult}}$$

13. The table is symmetric, so the person can sit near either edge and the same distance will result. We assume that the person (mass M) is on the right side of the table, and that the table (mass m) is on the verge of tipping, so that the left leg is on the verge of lifting off the floor. There will then be no normal force between the left leg of the table and the floor. Calculate torques about the right leg of the table, so that the normal force between the table and the floor causes no torque. Counterclockwise torques are taken to be positive. The conditions of equilibrium for the table are used to find the person's location.



$$\sum \tau = mg(0.60 \text{ m}) - Mgx = 0 \rightarrow x = (0.60 \text{ m}) \frac{m}{M} = (0.60 \text{ m}) \frac{24.0 \text{ kg}}{66.0 \text{ kg}} = 0.218 \text{ m}$$

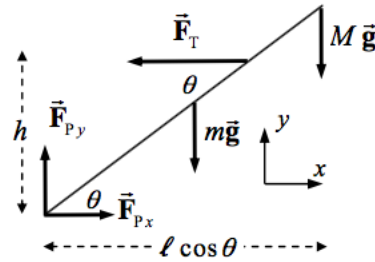
Thus the distance from the edge of the table is $0.50 \text{ m} - 0.218 \text{ m} = \boxed{0.28 \text{ m}}$.

21. (a) The pole is in equilibrium, and so the net torque on it must be zero. From the free-body diagram, calculate the net torque about the lower end of the pole, with counterclockwise torques as positive. Use that calculation to find the tension in the cable. The length of the pole is ℓ .

$$\sum \tau = F_T h - mg(\ell/2) \cos \theta - Mg\ell \cos \theta = 0$$

$$F_T = \frac{(m/2 + M)g\ell \cos \theta}{h}$$

$$= \frac{(6.0 \text{ kg} + 21.5 \text{ kg})(9.80 \text{ m/s}^2)(7.20 \text{ m}) \cos 37^\circ}{3.80 \text{ m}} = 407.8 \text{ N} \approx \boxed{410 \text{ N}}$$



- (b) The net force on the pole is also zero since it is in equilibrium. Write Newton's second law in both the x and y directions to solve for the forces at the pivot.

$$\sum F_x = F_{Px} - F_T = 0 \rightarrow F_{Px} = F_T = \boxed{410 \text{ N}}$$

$$\sum F_y = F_{Py} - mg - Mg = 0 \rightarrow F_{Py} = (m + M)g = (33.5 \text{ kg})(9.80 \text{ m/s}^2) = \boxed{328 \text{ N}}$$

32. Write the conditions of equilibrium for the ladder, with torques taken about the bottom of the ladder, and counterclockwise torques as positive.

$$\sum \tau = F_W \ell \sin \theta - mg\left(\frac{1}{2}\ell \cos \theta\right) = 0 \rightarrow F_W = \frac{1}{2} \frac{mg}{\tan \theta}$$

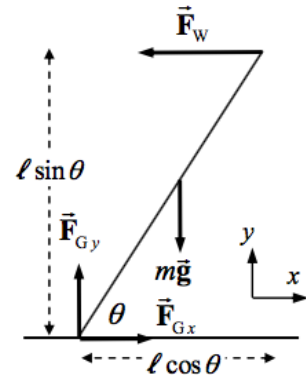
$$\sum F_x = F_{Gx} - F_W = 0 \rightarrow F_{Gx} = F_W = \frac{1}{2} \frac{mg}{\tan \theta}$$

$$\sum F_y = F_{Gy} - mg = 0 \rightarrow F_{Gy} = mg$$

For the ladder to not slip, the force at the ground F_{Gx} must be less than or equal to the maximum force of static friction.

$$F_{Gx} \leq \mu F_N = \mu F_{Gy} \rightarrow \frac{1}{2} \frac{mg}{\tan \theta} \leq \mu mg \rightarrow \frac{1}{2\mu} \leq \tan \theta \rightarrow \theta \geq \tan^{-1}\left(\frac{1}{2\mu}\right)$$

Thus the minimum angle is $\boxed{\theta_{\min} = \tan^{-1}\left(\frac{1}{2\mu}\right)}$.



34. The amount of stretch can be found using the elastic modulus in Eq. 12-4.

$$\Delta \ell = \frac{1}{E} \frac{F}{A} \ell_0 = \frac{1}{5 \times 10^9 \text{ N/m}^2} \frac{275 \text{ N}}{\pi (5.00 \times 10^{-4})^2} (0.300 \text{ m}) = \boxed{2.10 \times 10^{-2} \text{ m}}$$

35. (a) Stress = $\frac{F}{A} = \frac{mg}{A} = \frac{(25000 \text{ kg})(9.80 \text{ m/s}^2)}{1.4 \text{ m}^2} = 175,000 \text{ N/m}^2 \approx \boxed{1.8 \times 10^5 \text{ N/m}^2}$

(b) Strain = $\frac{\text{Stress}}{\text{Young's Modulus}} = \frac{175,000 \times 10^5 \text{ N/m}^2}{50 \times 10^9 \text{ N/m}^2} = \boxed{3.5 \times 10^{-6}}$

36. The change in length is found from the strain.

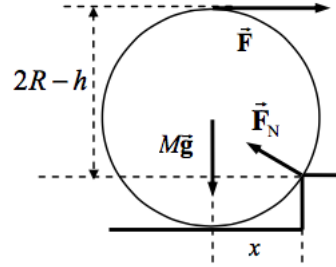
$$\text{Strain} = \frac{\Delta \ell}{\ell_0} \rightarrow \Delta \ell = \ell_0 (\text{Strain}) = (8.6 \text{ m})(3.5 \times 10^{-6}) = \boxed{3.0 \times 10^{-5} \text{ m}}$$

40. The percentage change in volume is found by multiplying the relative change in volume by 100. The change in pressure is 199 times atmospheric pressure, since it increases from atmospheric pressure to 200 times atmospheric pressure. Use Eq. 12-7.

$$100 \frac{\Delta V}{V_0} = -100 \frac{\Delta P}{B} = -100 \frac{199(1.0 \times 10^5 \text{ N/m}^2)}{90 \times 10^9 \text{ N/m}^2} = \boxed{-2 \times 10^{-2} \%}$$

The negative sign indicates that the interior space got smaller.

59. (a) If the wheel is just lifted off the lowest level, then the only forces on the wheel are the horizontal pull, its weight, and the contact force \vec{F}_N at the corner. Take torques about the corner point, for the wheel just barely off the ground, being held in equilibrium. The contact force at the corner exerts no torque and so does not enter the calculation. The pulling force has a lever arm of $R + R - h = 2R - h$, and gravity has a lever arm of x , found from the triangle shown.



$$x = \sqrt{R^2 - (R - h)^2} = \sqrt{h(2R - h)}$$

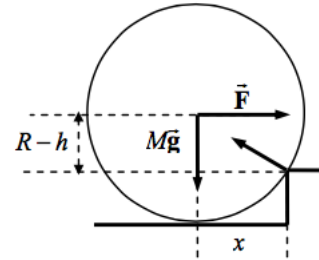
$$\sum \tau = Mg x - F(2R - h) = 0 \rightarrow$$

$$F = \frac{Mgx}{2R - h} = Mg \frac{\sqrt{h(2R - h)}}{2R - h} = \boxed{Mg \sqrt{\frac{h}{2R - h}}}$$

- (b) The only difference is that now the pulling force has a lever arm of $R - h$.

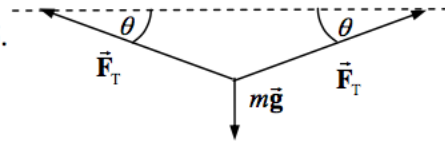
$$\sum \tau = Mg x - F(R - h) = 0 \rightarrow$$

$$F = \frac{Mgx}{R - h} = \boxed{Mg \frac{\sqrt{h(2R - h)}}{R - h}}$$



66. The radius of the wire can be determined from the relationship between stress and strain, expressed by Eq. 12-5.

$$\frac{F}{A} = E \frac{\Delta \ell}{\ell_0} \rightarrow A = \frac{F \ell_0}{E \Delta \ell} = \pi r^2 \rightarrow r = \sqrt{\frac{1}{\pi} \frac{F \ell_0}{E \Delta \ell}}$$

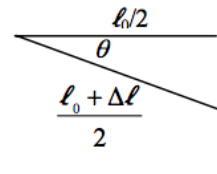


Use the free-body diagram for the point of connection of the mass to the wire to determine the tension force in the wire.

$$\sum F_y = 2F_T \sin \theta - mg = 0 \rightarrow F_T = \frac{mg}{2 \sin \theta} = \frac{(25 \text{ kg})(9.80 \text{ m/s}^2)}{2 \sin 12^\circ} = 589.2 \text{ N}$$

The fractional change in the length of the wire can be found from the geometry of the problem.

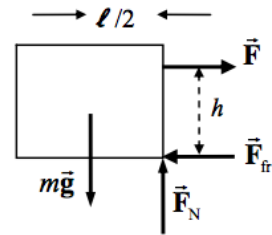
$$\cos \theta = \frac{\ell_0/2}{\frac{\ell_0 + \Delta \ell}{2}} \rightarrow \frac{\Delta \ell}{\ell_0} = \frac{1}{\cos \theta} - 1 = \frac{1}{\cos 12^\circ} - 1 = 2.234 \times 10^{-2}$$



Thus the radius is

$$r = \sqrt{\frac{1}{\pi} \frac{F_T \ell_0}{E \Delta \ell}} = \sqrt{\frac{1}{\pi} \frac{589.2 \text{ N}}{70 \times 10^9 \text{ N/m}^2} \frac{1}{(2.234 \times 10^{-2})}} = \boxed{3.5 \times 10^{-4} \text{ m}}$$

70. If the block is on the verge of tipping, the normal force will be acting at the lower right corner of the block, as shown in the free-body diagram. The block will begin to rotate when the torque caused by the pulling force is larger than the torque caused by gravity. For the block to be able to slide, the pulling force must be as large as the maximum static frictional force. Write the equations of equilibrium for forces in the x and y directions and for torque with the conditions as stated above.



$$\sum F_y = F_N - mg = 0 \rightarrow F_N = mg$$

$$\sum F_x = F - F_{fr} = 0 \rightarrow F = F_{fr} = \mu_s F_N = \mu_s mg$$

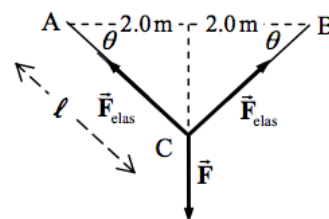
$$\sum \tau = mg \frac{\ell}{2} - Fh = 0 \rightarrow \frac{mg\ell}{2} = Fh = \mu_s mgh$$

Solve for the coefficient of friction in this limiting case, to find $\mu_s = \frac{\ell}{2h}$.

(a) If $\mu_s < \ell/2h$, then sliding will happen before tipping.

(b) If $\mu_s > \ell/2h$, then tipping will happen before sliding.

98. See the free-body diagram. We assume that point C is not accelerating, and so the net force at point C is 0. That net force is the vector sum of applied force \vec{F} and two identical spring forces \vec{F}_{elas} . The elastic forces are given by $F_{\text{elas}} = k(\text{amount of stretch})$. If the springs are unstretched for $\theta = 0$, then 2.0 m must be subtracted from the length of AC and BC to find the amount the springs have been stretched. Write Newton's second law for the



vertical direction in order to obtain a relationship between F and θ . Note that $\cos \theta = \frac{2.0 \text{ m}}{l}$.

$$\sum F_{\text{vert}} = 2F_{\text{elas}} \sin \theta - F = 0 \rightarrow F = 2F_{\text{elas}} \sin \theta$$

$$F_{\text{elas}} = k(\ell - 2.0 \text{ m}) = k\left(\frac{2.0 \text{ m}}{\cos \theta} - 2.0 \text{ m}\right) \rightarrow$$

$$F = 2F_{\text{elas}} \sin \theta = 2k\left(\frac{2.0 \text{ m}}{\cos \theta} - 2.0 \text{ m}\right) \sin \theta = 2(20.0 \text{ N/m})(2.0 \text{ m})\left(\frac{1}{\cos \theta} - 1\right) \sin \theta$$

$$= 80 \text{ N}(\tan \theta - \sin \theta)$$

This gives F as a function of θ , but we require a graph of θ as a function of F . To graph this, we calculate F for $0 \leq \theta \leq 75^\circ$, and then simply interchange the axes in the graph.

The spreadsheets used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH12.XLS", on tab "Problem 12.98".

