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4B FINAL

Look over the entire Final before starting with the easiest problems first.

This Final is closed book. Your blue book should be blank and contain no information. Do not just give answers from memory but show where they come from by showing work.
Make large and neat figures. Make sure to always indicate positive directions. Specify the units of numerical answers.

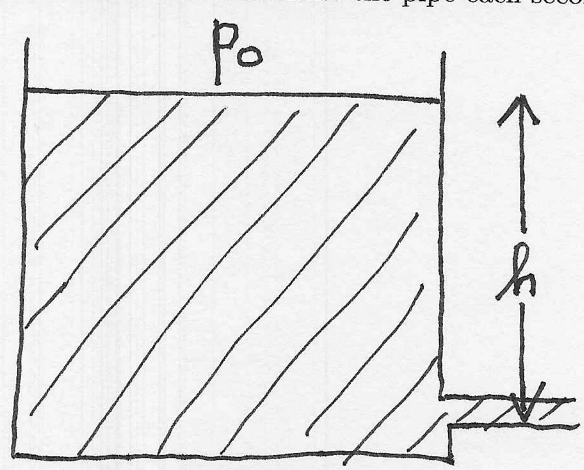
Some constants: $1\,\mathrm{kcal} = 4.2\,\mathrm{kJ}$, $c(\mathrm{ice}) = 0.5\,\mathrm{kcal/(kg\,°C)}$, $R = 8.3\,\mathrm{kJ/(kmol\,°K)}$, $k = 1.4 \times 10^{-23}\,\mathrm{J/kg}$, $N_A = 6.0 \times 10^{26}\,\mathrm{kmol^{-1}}$, $1\,\mathrm{u} = 1.7 \times 10^{-27}\,\mathrm{kg}$, $\hbar = 6.6 \times 10^{-34}/(2\pi)\,\mathrm{J\,sec}$. If you need other constants please ask.

PROBLEM 1 (15 points)

A vessel filled with water is emptied through a thin pipe with area A at its bottom, see the Figure. The vessel is so big that the outflow of water does not affect the height h of the water above the center of the pipe. The atmospheric pressure is p_0.

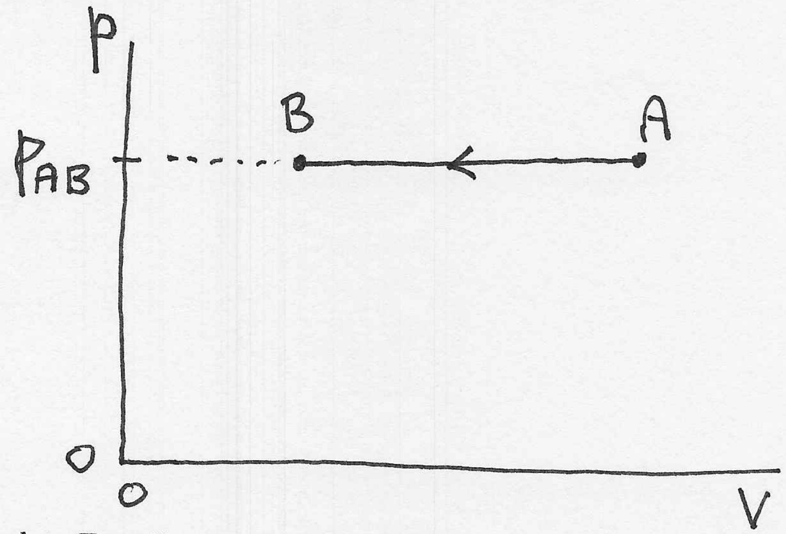
a. What is the velocity of the water as it exits the pipe?

b. What is the mass of the water that exits the pipe each second?



PROBLEM 2 (25 points)

An ideal gas is brought from A to B as shown in the p, V diagram. The process is reversible.



a. Calculate $T_B - T_A$ and T_B/T_A.

b. Calculate how much heat is exchanged and specify clearly in which direction.

c. How much work is done? Specify whether the work is done by the gas on the environment or by the environment on the gas?

d. Indicate in the p, V diagram how the amount of work being done can be graphically represented and how that graphic shows the answers (plural) to c).

d. Calculate the change in entropy $S_B - S_A$.

e. Is the change positive or negative and why?

f. Make a *rough* sketch of the path in a T, S diagram showing the letters A and B.

g. What is the change in the total entropy of the gas and its environment? Why?

PROBLEM 3 (20 points)

A heating element in a water heater supplies 2.0 kW of power. It contains 0.5 kG of water that is initially at 20 °C. The water heater is a cheap one that loses heat at the rate of 50 W per each degree Celcius temperature difference between the water and the 20 °C environment.

a. What will be the eventual temperature after a long time?

b. If the power is increased from 2.0 kW to 5.0 kW how long will it take for the water to completely evaporate, calculating the time from the moment that the water started to boil?

PROBLEM 4 (20 points)

An oscillator of mass m is initially (at $t = 0$) at position $x = x_0$. Its velocity is given by $v(t) = v_0 \cos(2\pi f t)$ where v_0 is a constant.

a. What is the physical meaning of the quantities v_0 and f?

b. What are the period and the angular frequency of the motion?

c. What is the acceleration of the motion as function of time?

d. What is the position as function of time?

e. What is the kinetic energy as function of time?

f. What is the average kinetic energy?

g. What is the phase difference between position and velocity?

PROBLEM 5 (30 points)

A string executes transverse oscillations $y(x,t)$ specified by the wave equation

$$\frac{\partial^2 y}{\partial t^2} - \frac{T_s}{\mu}\frac{\partial^2 y}{\partial x^2} = 0 \tag{1}$$

where T_s is the tension in the string and μ its mass per unit length.

a. Specify the properties of the differential equation (partial or ordinary, linear or not, order, homogeneous or not, constant coefficients or not).

b. How many independent solutions do you expect and why?

c. Find both solutions.

d. Define all constants that appear in your solutions in terms of T_s and μ.

e. Specify the wavelength, wave number, frequency, angular frequency, and propagation velocity in terms of T_s and μ.

PROBLEM 6 (25 points)

A longitudinal wave is propagating in a thin metal bar. The Figure shows a small piece of it prior to the passing of a wave. Its area is A, it is positioned with its left side at x and its right side at $x + dx$, dx is infinitely small. The bar's (volume) mass density is ρ_0. When a longitudinal wave passes through the slab, the displacement of atoms in the bar from their equilibrium position at position x and time t is given by $s(x, t)$.

a. What are the volume and mass of the slab prior to the wave passing through it?

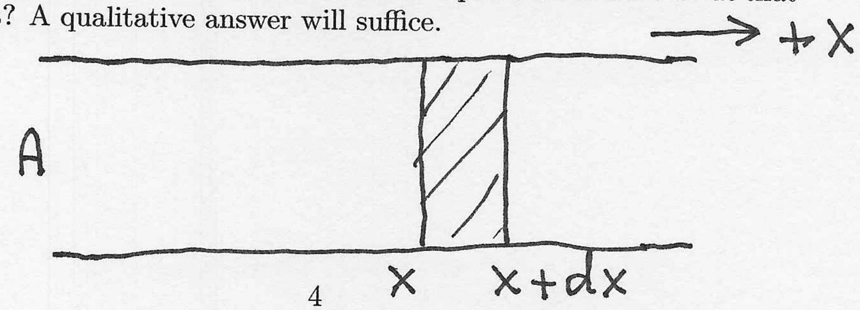
b. What is the position of the left side and the right side of the slab at time t as the wave passes through the slab?

c. What is the new volume of the slab when the wave passes through it?

d. Which quantity is unchanged (conserved) as the wave passes through the slab?

e. Use the conservation property of d) to relate the prior density ρ_0 to the new density ρ that is present when the wave passes through the slab.

f. If $\partial s(x,t)/\partial x < 0$ what happens to the temperature of the slab at that instant? A qualitative answer will suffice.



PROBLEM 7 (15 points)

A propagating wave has its power reduced by 20 dB and later once more by 30 dB. By how many dB is to power finally reduced? Explain.

PROBLEM 8 (25 points)

A source S of light waves is moving with constant velocity v_S relative to observer A who is stationary. The source S produces waves with frequency f as observed by an observer B who is at rest relative to the source. The propagation velocity of the waves is c (the velocity of light). The first detected beat of the light waves took a time $t = L/c$ to reach observer A where L is the distance between observer A and the source S when the first detected beat was emitted by the source.

a. What is the distance between the source S and observer A when the second beat is emitted by the source S?

b. How much time does it take for the second beat to travel from the source S to observer A?

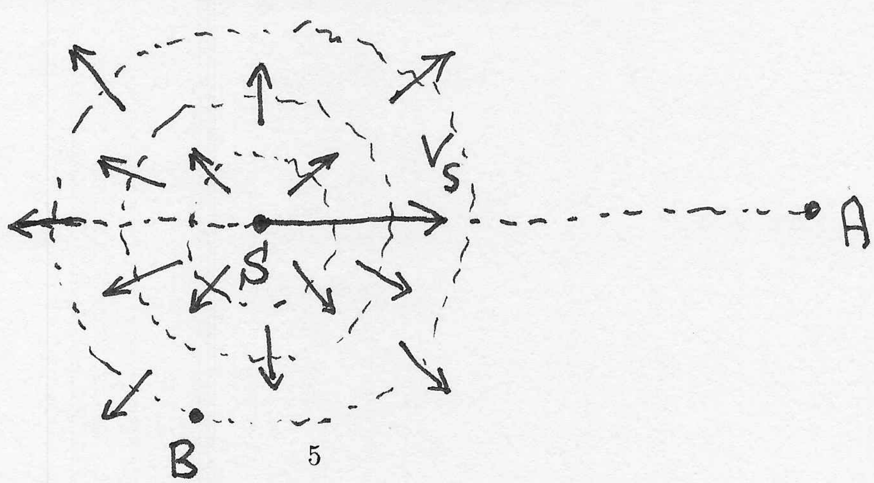
c. Same two questions for the third beat.

d. How much time elapses between the detection of successive beats?

e. What is the frequency f' of the light waves detected by observer A?

f. The radiation from a far away galaxy is detected with a frequency $f' = 1.5f$. What is the velocity of the galaxy relative to observer A?

g. Is the galaxy moving toward or away from observer A?



PROBLEM 9 (20 points)

The kinetic theory of ideal gases gives $pV = \frac{1}{3}Nm\langle v^2 \rangle$.

a. What is N in this formula?

b. Is the formula correct only for mono-atomic gases or does it also apply to gases that consist of molecules? Explain your answer.

c. Is $\langle v^2 \rangle = \langle v \rangle^2$? Why (not)? Give a simple example to show this.

d. Calculate v_{rms} as function of the temperature T.

e. Calculate the average kinetic energy per molecule from its translational motion.

PROBLEM 10 (20 points)

At temperatures well above room temperature solids are observed to have a molar specific heat capacity of $3R$ with R the *gas* constant. Explain this (the factor 3, why it is the same for all solids, and how it can be that a gas constant enters in an expression for the molar specific heat of a *solid*).

PROBLEM 11 (30 points)

Gas molecules are forced to move along one direction only: the x-axis. The temperature is T.

a. What is the probability for v_x to be in the interval $[v_x, v_x + dv_x]$?

b. Using the result from a) calculate $\langle v_x \rangle$.

c. Write the formula for $\langle v_x^2 \rangle$.

d. Do the integral for extra credit (20 points). You will need

$$\int_{-\infty}^{+\infty} \mathrm{e}^{-\alpha x^2} dx = \sqrt{\frac{\pi}{\alpha}} \tag{2}$$

and its derivative with respect to α.

e. Is $\langle v_x \rangle^2 = \langle v_x^2 \rangle$? Explain.

f. What do you expect $\frac{1}{2}m\langle v_x^2 \rangle$ to be equal to?

g. If you did the integral in c) for extra credit do you find the answer in f)?

PROBLEM 12 (25 points)

A solid has a molar specific heat capacity $c = a/T$ where T is its temperature and a is a constant. Heat is transfered to the solid such that its temperature increases from T_1 to T_2. The solid contains n kilomols, its thermal expansion can be neglected.

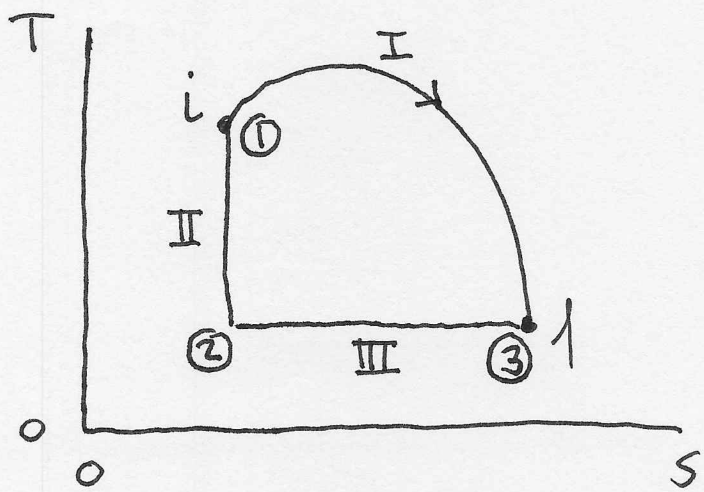
a. How much work is done by the solid on the external world during the temperature change?

b. Calculate the change in entropy due to the temperature change.

c. Make a plot of the entropy as function of temperature showing the change. Clearly label the start and end of the path executed by the solid.

PROBLEM 13 (40 points)

An ideal gas is brought from an initial state i to a final state f using a reversible process. The T, S diagram shows the path I that is followed by the system. We want to calculate the difference in entropy $S_f - S_i$. Because the path is complicated it is decided to calculate $S_f - S_i$ using a different reversible path indicated by II and III.



a. Is this allowed? If so, why?

b. Name path II and calculate the change in entropy $S_2 - S_1$ along it.

c. Name path III and calculate the change in entropy $S_3 - S_2$ along it.

d. Calculate $S_f - S_i$.

e. Is any heat exchanged between the gas and its environment? If so specify magnitude and direction of the heat exchange.

f. Indicate in the T, S diagram how the amount of heat being exchanged can be graphically represented.

PROBLEM 14 (30 points)

A quantum harmonic oscillator has discrete energy levels as discussed in class. Its fundamental vibration frequency is ω_0 and it is in equilibrium with its environment at temperature T.

a. Calculate the probability to find the oscillator at its ground state (the state with lowest energy).

b. The hydrogen molecule has a fundamental vibration frequency $\omega_0 = 1.3 \times 10^{14}$ Hz. Calculate the probability in a) when $T = 300\,°\text{K}$.

c. Calculate the probability in a) when $T = 1\,°\text{K}$.

d. Comment of your results of b) and c): What is the effect of temperature on the quantum oscillator?