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DIFFERENTIAL EQUATIONS

I discuss nomenclature in characterizing Differential Equations (DEs) with a few remarks about solving them. This will be done by examples. The generalization to the general case will be left to Math 20D course.

Consider

$$F_t \frac{\partial^2 D}{\partial x^2} = \mu \frac{\partial^2 D}{\partial t^2} \quad (1)$$

This DE can be found in Giancoli P.407 This equation has x and t as independent variables and $D = D(x, t)$ as dependent variable. Here F_T and μ are constants. Because the DE involves partial differentials it is called a partial DE. Because the DE does *not* have a term independent of D it is called a homogeneous DE. An example of an inhomogeneous DE is

$$F_t \frac{\partial^2 D}{\partial x^2} = \mu \frac{\partial^2 D}{\partial t^2} + A \sin \omega t \quad (2)$$

where A is a constant. Note that the criterion for an homogeneous DE is *not* that the right-hand side of the DE is zero. That can always be accomplished for an inhomogeneous DE by moving all terms to the left and doing that should not make an inhomogeneous DE into a homogeneous DE.

Eq.(1) and (2) have constant coefficients (F_T , μ , and A). This does not have to be so. F_T , μ , and A could be functions of x and t and the DEs would still be called partial DEs that are homogeneous for Eq.(1) and inhomogeneous for Eq.(2).

Eq.(1) and (2) are of second order because the highest derivative is a second derivative. This does not have to be so, any order is permitted. An example of a first order partial DE is

$$F_t \frac{\partial D}{\partial x} = \mu \frac{\partial D}{\partial t} + A \sin \omega t \quad (3)$$

All above DEs are linear DEs because all of the terms involving the dependent variable D and its derivatives are raised to the first power. An example of a non-linear DE is

$$F_t \left[\frac{\partial^2 D}{\partial x^2} \right]^2 = \mu \frac{\partial^2 D}{\partial t^2} + A \sin \omega t \quad (4)$$

This DE is still of second order but is called quadratically non-linear. The next equation is linear even though it has a term raised to the third power

$$F_t \frac{\partial^2 D}{\partial x^2} = \mu \frac{\partial^2 D}{\partial t^2} + [A \sin \omega t]^3 \quad (5)$$

This is because that term does not involve the dependent variable D but only the independent variable t .

DEs that have no partial derivatives are called normal DEs. We encountered three examples in Giancoli Ch.14. An example is provided by the DE for a driven oscillatory motion without friction

$$m \frac{d^2 x}{dt^2} + kx = F_0 \cos(\omega t) \quad (6)$$

This is an ordinary DE, linear, of second order, inhomogeneous, with constant coefficients.

We will concentrate in Physics 4B on DEs that are linear with constant coefficients and not higher than second order. They are either homogeneous or inhomogeneous. These are DEs that keep coming back in different areas of the sciences. Fortunately we know how to solve them analytically.

The number of solutions of these DEs is equal to the order of the DE. To solve an inhomogeneous second order DE you must follow a two step program in any order: Solve the DE as given to get one solution. Then remove the term(s) that make the DE inhomogeneous and solve the homogeneous version. The general solution of the DE is the sum of the two solutions each multiplied by an arbitrary constant.

All this will be proven in Math 20D.