4A QUIZ #4

This Quizz is closed book.

PROBLEM 1 (35 points).

A particle of mass \( m \) performs a one dimensional linear motion along the \( x \)-axis. It is initially positioned at \( x = 0 \) with a velocity \( v \). It is subject to a potential energy \( U(x) \) that has the form of a sawtooth as shown in the figure. The slopes of the teeths are 45 deg and the peaks closest to the particle are at \( x = \pm a \) with heights \( U_0 \).

![Diagram of sawtooth potential energy function](image)

a. How far from \( x = 0 \) will the particle be able to move if its kinetic energy \( K \) is less than \( U_0 \)?

b. Describe the motion of the particle under the condition mentioned in a).

c. Now assume that the particle’s kinetic energy \( K \) is larger than \( U_0 \). Describe qualitatively the motion of the particle under this condition.

d. Calculate the velocity of the particle at the first peak of the sawtooth
under the condition mentioned in c).

e. What is the velocity of the particle at the other peaks of the sawtooth? Explain.

PROBLEM 2 (30 points).
A homogeneous plate of material with constant thickness $D$ has a shape and dimensions as shown in the Figure. Find the position of the center-of-mass in three dimensions. Specify your choice of the location of $z = 0$.

PROBLEM 3 (35 points).
A person weighing 100 kg is standing at the edge of a flat car that is 10’ long as shown in the Figure. The car weights 200 kg.
a. Calculate the position of the center-of-mass of the combination of car and person measured along the length of the car. Be clear about your coordinate system when you specify the location of the center-of-mass.

b. The person moves from one end of the car to the other end. Does the car move and if so by how much? Be clear about your reasoning.

c. If the person walks to the other side of the car at a constant speed of \(1 \text{ m/sec}\) (as measured by a coordinate system at rest on the ground) calculate the velocity of the cart (magnitude and direction) during the walk.

Extra credit (30 points).

A solid object consists of two identifiable parts with mass \(m_1\) and \(m_2\) respectively. One part has its center-of-mass at \(\vec{r}_1\) and the other part has it at \(\vec{r}_2\). Assuming that \(m_1, m_2, \vec{r}_1,\) and \(\vec{r}_2\) are given, find an expression for the location of the center-of-mass of the object.